

# Stoic Logic and Multiple Generality

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## Introduction

Multiple generality is the existence in a sentence or predicate of one quantifier in the scope of another. An example of a sentence with multiple generality is

Everyone loves someone.

With  $Fxy$  for the predicate 'x loves y', one standard symbolization is

$\forall x\exists y Fxy$

The number of possible nestings of quantifiers in the scope of quantifiers is unlimited. Certain basic natural language inferences require multiple generality. For example, with  $a$  for Dio and  $b$  for Plato:

Everyone loves everyone.  $\forall x\forall y Fxy$

Hence Plato loves Dio.  $Fab$

Everyone loves everyone.  $\forall x\forall y Fxy$

Hence everyone loves someone.  $\forall x\exists y Fxy$

It is generally agreed that one of Frege's core achievements was the development of a logic that can account for multiple generality and that for this purpose he instituted rules that govern the stacking of quantifiers.<sup>1</sup> We don't quibble with this. There is no explicit surviving evidence that the Stoics had a fully worked-out theory of multiple generality. Instead, we argue that the Stoics had all the elements required to introduce multiple generality. More precisely, that among the sparsely surviving evidence on Stoic logic there is sufficient material to establish that the Stoics had all those elements for existential and universal quantification with more than one quantifier, if not exactly in the way Frege introduced them. Rather, their system of quantification is variable-free, not unlike that introduced by Quine in 1960.<sup>2</sup> To this end,

1. Frege 1879, §11. See Dummett 1973, 9; Rumfitt 1994, 599–607; Zalta 2018.
2. Quine 1960, cf. 1971, 1981. See also the work by Pauline Jacobson, e.g. Jacobson 1999.

and building on existing literature,<sup>3</sup> we consider and reinterpret some passages that have not yet been given much attention in the context of multiple generality and specify how multiple generality played an active role in Stoic logic. (We note that the surviving evidence of Stoic logic of predicates covers only a tiny fraction of what we know the Stoics wrote on the topic.)

The relevance of our undertaking is threefold. First, it establishes that Stoic logic, though mostly forgotten at the beginning of the Middle Ages, if not already in the fourth century CE, sports considerable advantages over Aristotle's logic in expressing and dealing with multiple generality. Second, it indicates that from Aristotle to Frege, instead of one big step — a logical discovery to which all intervening philosophers were 'simply blind' (Dummett 1973, 9) — there is a somewhat more gradual development. (We remark on the relation between Stoic logic and medieval attempts at figuring out multiple generality briefly in our conclusion.) Third, it is a reminder that the focus on multiple generality exclusively through the lens of Frege-inspired variable-binding quantifier theory — as contrasted with variable-free predicate logic — may prevent our appreciation of the development of pre-Fregean theories of multiple generality. Additionally, we offer new explanations of a couple of puzzling elements in Stoic logic.

One goal of the paper is to introduce a larger audience to the intricacies of Stoic logic and to its more general potential. It is for this reason that we very occasionally add a remark about how the Stoic theory could be extended in an obvious manner to cover more general cases for which there is no evidence either way. Two examples are polyadic predicates with higher argument numbers and unrestricted

3. Some excellent work has been done on Stoic predicates (including some on monadic quantification). We mention in particular Atherton 1993, 44–48, 259–264; Atherton & Blank, 2003, 314–316, 320–323; Barnes 1986, Barnes 1999; Barnes et al. 1999, 111–114, 197–206; Brunschwig 1986, 287–310 = 1994, 63–67; Crivelli 1994a, 189–199; Frede 1974, 51–73; Gaskin 1997, 91–104; Lloyd 1978. There are also most useful observations in Durand 2018; Egli 2000; Long & Sedley 1987, v.1, 199–200; Hülser 1987–88; Hülser et al. 2009. Stoic polyadic quantification has been considered by Urs Egli in his 1993 and 2000, and we agree with some of his results.

universals. We hope that the footnotes will satisfy the expectations of those specializing in Stoic logic that no historical or methodological corners have been cut.

The paper is structured as follows: §1 Relevant general remarks on Stoic logic; §2 Stoic *katêgorêmata* as monadic predicates; §3 Monadic predicates as functions; §4 Polyadic predicates; §5 Variable-free quantification I: monadic indefinite propositions; §6 Variable-free quantification II: polyadic indefinite propositions; §7 Multiple generality I: scope ambiguity; §8 Multiple generality II: anaphoric ambiguity; §9 Polyadic predicates and Stoic deduction; §10 Concluding remarks.

### 1. Some general remarks on Stoic logic

We remind readers of some very basic elements of Stoic logic. The Stoics sharply distinguish linguistic items or 'speech' (*logos*) from what speech signifies. Speech is a species of sound, namely, sound that signifies meaning or is significant (*sêmantikê*) (DL 7.55–56, 63). 'Sayables' (*lekta*), by contrast, are the incorporeal items that are signified by speech (DL 7.57). For example, in uttering "Plato walks", a speaker 'says' the sayable «Plato walks». As what is signified by speech, sayables are *contents* of speech. They thus play a role analogous to Fregean senses.<sup>4</sup> We use double quotation marks to indicate linguistic items (speech), guillemets («, ») to indicate content (sayables). Stoic grammar studies the properties and parts of *speech*, while Stoic logic studies the properties and parts of *sayables* (DL 7.43–44, 63).<sup>5</sup> Although many later ancient sources conflate this distinction, orthodox Stoics are careful to keep apart the subject-matter of grammar and logic.

Stoic contents are structured, and their structure corresponds — to some degree — to the structure of language. In classifying the various kinds of content, the Stoics rely on grammatical properties of the linguistic items that express them. For instance, the monadic predicate (*katêgorêma*) «...loves Plato» is signified by a verb and a declined noun,

4. Gaskin 1997, 94–95; Barnes et al. 1999, 95–96.

5. For an excellent, detailed introduction to Stoic grammar, see Atherton & Blank 2003. An excellent introduction to Stoic logic is Ierodiakonou 2006.

and Stoic propositions (*axiōmata*) tend to be signified by declarative sentences. Propositions are the fundamental items within Stoic logic and, as the sole non-derivative bearers of truth-value, can be compared to Fregean thoughts.<sup>6</sup> So the relation between sayables and propositions is analogous to that between Fregean senses and Fregean thoughts. The most basic distinction of Stoic propositions is that between simple and non-simple ones (DL 7.68–69; SE M 8.93, 95, 108). The simple ones include negations of *simple* propositions. Non-simple propositions are those that are put together from more than one proposition or one proposition taken twice and that are governed by one or more connective parts or a negation operator. For example, a disjunction is governed by the connective parts ‘either’ (for the first disjunct) and ‘or’ (for the second). The principal non-simple propositions are conjunction, conditional and exclusive disjunction.<sup>7</sup> Negations and non-simple propositions are defined iteratively: the language of Stoic propositional logic is syntactically closed under negation, conjunction, disjunction and conditional.

The Stoics do not posit a perfect one-to-one correspondence between content and speech. The grammatical properties of speech are a defeasible, and potentially misleading, guide to the content it signifies. (“*p* and *q* or *r*” is an example.) On the Stoic view, one and the same expression of natural language, if it is ambiguous, expresses multiple contents.<sup>8</sup> Moreover, the same content can be signified by different pieces of speech (Barnes et al. 1999, 96–97).

6. See Barnes et al. 1999, 93–96, for the limitations of this comparison. Note also that there is disagreement among scholars on Stoic logic and linguistics with regard to the question whether sayables are mind-independent. Mind-dependency is defended most recently in de Harven 2018, 228–230 and before that e.g. in Long 1971, 96–98, while e.g. Barnes 1999, 211; Shogry 2019, 37 fn. 12; and Bronowski 2019, 165–169, defend mind-independency. This paper is independent of how one leans on this question.
7. DL 7.68–74, SE M 8.93–94, 108. For detailed treatment of the simple and non-simple propositions, see Barnes et al. 1999, 96–111.
8. For detailed discussion, see all of Atherton 1993, but esp. 131–133.

To remove ambiguity in natural language, the Stoics introduce a system of linguistic conventions that ensure that the form of speech reveals the contents being expressed.<sup>9</sup> Many of them concern word order. Most languages have either case marking or rigid order (Miyagawa 2012, ch. 10). Whereas English, for example, has basically no case marking but fairly rigid order, ancient Greek is found toward the other end of the spectrum, with very little rigid order but fairly articulated case marking. This works to the advantage of the Stoics in their attempt to structurally disambiguate language by means of regimentation. It is far easier to introduce some distinctive requirements of rigid order into a natural language with extensive case marking than to introduce case markings into a natural language with rigid order. For instance, to signify the negation of «Plato is walking», the formulation “Plato is not walking” is discouraged by the Stoics. It is reserved for the affirmation «Plato is not walking», which, since it is assumed to entail the existence of Plato, is not contradictory to «Plato is walking».<sup>10</sup> Instead, to express negations the Stoics recommend prefixing the negation particle to the sentence that signifies the proposition it is negating thus “Not: Plato is walking” (which is grammatical in Greek). This accurately reflects the scope of the Stoic negation operator. Generally, there is plentiful evidence that the Stoics used the following principle, which we call the

**Scope Principle:** The expression that signifies the content element or operator with the largest scope in a proposition is the first expression in the sentence, or as close to the beginning of the sentence as grammar permits. If an operator consists of more than one part, the expression that signifies its first part is the first expression in the

9. On Stoic natural language regimentation, see Bronowski 2019, 212–214; Atherton & Blank 2003, 314–316; Barnes et al. 1999, 96–97; and Frede 1974.
10. See e.g. Apul. *Herm.* 191.6–11, Alex. *An. Pr.* 402.3–19 and Barnes et al. 1999, 102, and also §7 below.

sentence, or as close to the beginning of the sentence as grammar permits.<sup>11</sup>

In the Stoic view, language (speech), suitably regimented, is an appropriate tool to represent the structure of sayables, i.e. of content.

Stoic propositions are of central importance for the Stoic system of deduction (Stoic syllogistic), which, unlike Aristotle's syllogistic, is a propositional sequent logic. This notwithstanding, the Stoics, in particular Chrysippus, third head of the Stoa and by far the greatest Stoic logician, displayed a keen interest in the logical significance of sub-propositional elements. These include (i) the logical relations between 'says that *x*', '*x*' and '*x* is true', possibly in connection with the Liar paradox; (ii) a sophisticated theory of demonstratives; (iii) the logic of plural expressions;<sup>12</sup> and (iv), most relevant to our purposes, a basic system of variable-free quantifying operators and a detailed classification of predicate contents.

11. Cf., for example, the Stoic definitions and examples of negative proposition (ἀποφατικόν, starting with οὐχί: DL 7.69; SE M 8.89, 8.103; cf. Apul. *Herm.* 191.6–11 *cui negativa particula praeponitur*), eliminating proposition (ἀρνητικόν, starting with οὐδέεις: DL 7.70), privative proposition (στερητικόν, starting with predicate expression with *alpha privativum*: DL 7.70), affirmative proposition (κατηγορικόν, starting with noun/name: DL 7.70), middle proposition (μέσσο, starting with noun/name: SE M 8.97), deictic proposition (καταγορευτικόν, starting with demonstrative pronoun: DL 7.70), definite proposition (ὠρισμένον, starting with demonstrative pronoun: SE M 8.96–97, Alex. *An. Pr.* 177–178), conditional proposition (συνημμένον, starting with εἰ: DL 7.71, SE PH 2.157–158, SE M 8.109–110), paraconditional (παρασυνημμένον, starting with ἐπεὶ: DL 7.71), conjunctive proposition (συμπελεγμένον, starting with καί: DL 7.72, Apoll. *Dysc. On Conjunctions* 218.15–19), negated conjunctive proposition (ἀποφατική συμπλοκή, starting with οὐχί καί: SE M 8.226), disjunctive proposition (διεzeugμένον, starting with ἢτοι or ἢ: DL 7.72; SE PH 1.69, 2.158; SE M 8.434; cf. Gellius 16.8.13), causal proposition (αἰτιώδες, starting with διότι: DL 7.72), co-assumption (πρόσληψις, has particle δέ as second word: DL 7.76), conclusion (ἐπιφορά, has ἄρα as second word: DL 7.76, SE M 8.302), question (ἐρώτημα, starting with ἄρα: DL 7.66, Ammon. *Int.* 199.20–23), inquiry (πύσμα, starting with an interrogative pronoun, e.g. ποῦ: SE 8.71–72, Ammon. *Int.* 200.5–10), quasi-proposition (ἴμοιον ἀξιωματι, starting with ὡς: DL 7.67, Ammon. *Int.* 2.32–34). See also Barnes et al. 1999, 101; Atherton 1993, 78–79; Frede 1974, 189–201.
12. See (i) Crivelli 1994b; (ii) Frede 1974, 53–61; Lloyd 1978; Barnes et al. 1999, 93–101; Durand 2018, 103–131; (iii) Cavini 1993; Barnes et al. 1999, 152–155.

Work on Hellenistic philosophy is methodologically complex, and work on Stoic logic is so in particular. Evidence is very fragmentary. Of hundreds of books (i.e. papyrus rolls) on Stoic logic, only one has survived (Chrysippus' *Logical Investigations*),<sup>13</sup> and in a sorry state. Everything else is one or more steps removed from the original texts. Sources are dependable to different extents for various reasons, and we will occasionally remark on the reliability of a source. For details, the reader is referred to specialist secondary literature.<sup>14</sup> Some guidance is given by extensive lists of book titles on Stoic logic and a detailed summary of Stoic logic in Diogenes Laertius. Many passages of great interest have survived quoted or paraphrased by often much later authors, many of them hostile to Stoic philosophy. We do our best to unscramble the scraps of egg. Translations are our own, unless otherwise noted.

## 2. Stoic katêgorêmata as monadic predicates: definitions

Stoic predicates (their term is *katêgorêma*, plural *katêgorêmata*) are contents<sup>15</sup> and as such belong to Stoic logic, rather than Stoic grammar. Our sources indicate that, probably starting with Chrysippus, the Stoics had an elaborate logical theory of katêgorêmata that was developed over several generations.<sup>16</sup> Chrysippus', and perhaps generally

13. PHerc. 307, col. xiii.19–22, ed. L. Marrone, 'Le *Questioni Logici* di Crisippo (PHerc 307)' in *Cronache Ercolanesi* 27, 1997, 83–100. A lacunose but long papyrus fragment and the only one of Chrysippus' books on logic of which we have direct evidence. For excellent general discussion of this text, see Barnes 1986; Barnes et al. 1999, 69–71; and Marrone 1997. Earlier editions of the papyrus appear in FDS as 698 and in SVF 2 as *frag.* 298a.
14. E.g. Mansfeld 1999, 3–30; Hülser 1987–88, XXXII–LXVIII; Bobzien 1998, 5–12; Barnes 1999, 69–76.
15. The first Stoic for whom there is evidence for this view is Chrysippus' predecessor Cleanthes, who states that katêgorêmata are *lekta* (Clement, *Strom.* 8.9.26.4). Post-Cleanthean evidence for this claim is adduced throughout this Section.
16. Chrysippus wrote fourteen books on katêgorêmata, one on active (*ortha*) and passive (*huptia*) katêgorêmata, one on event-predicates (*sumbamata*, emendation, von Arnim) (DL 7.191). We expect treatment of katêgorêmata in his books on indefinite and temporal propositions (DL 7.190). Katêgorêmata

the early Stoic, notion of *katêgorêmata* was one of *monadic* predicates. A matching Stoic definition is

(A) A *katêgorêma* is ... an incomplete sayable that can be connected with an upright case-content (*orthê ptôsis*) to yield a proposition.<sup>17</sup> (DL 7.64)

The definition (A) classifies *katêgorêmata* as a kind of content that can be connected with other things. It specifies these things as upright case-contents, and the resulting content as a proposition. A Stoic *case-content* (*ptôsis*) can be thought of as the content signified by a noun.<sup>18</sup> An *upright case-content* is the case-content signified by a *nominative* noun. Thus text (A) suggests that a *katêgorêma* is akin to a monadic predicate, except that contemporary theories do not require that the analogue to Stoic case-content be in the nominative (see also below §3). Two later ancient texts contain variants of this definition.

(B) Now, if something is predicated of a noun and yields an assertible content, it is called by them [i.e. the Stoics]

also feature prominently in Chrysippus' *Log. Inv.* Apollodorus offered a definition of *katêgorêma* (below). Cleanthes and Sphaerus of Borysthenes (pupil of Zeno and Cleanthes) authored one work each on *katêgorêmata* (DL 7.175, 178), though these *may* concern their causal aspect. For these aspects of *katêgorêmata*, see Bobzien 1998, 18–21, and Hankinson 1999, 483–486. We set them aside here.

17. Ἔστι δὲ τὸ κατηγορήμα ... λεκτὸν ἔλλιπές συντακτὸν ὀρθῇ πῶσει πρὸς ἀξιώματος γένεσιν. We here translate *suntaktos* as 'can be connected', rather than the alternative 'connected', since a *katêgorêma* is not necessarily always connected with something (DL 7.63 and §3). (A) is the third in a list of three independent definitions, most probably originating in different works by different Stoics, as is common in compendia and epitomes of Stoic doctrine. The first definition is 'a *katêgorêma* is that which is said of something' ('Ἔστι δὲ τὸ κατηγορήμα τὸ κατὰ τινος ἀγορευόμενον'). It is less specific than (A), but indicates that *katêgorêmata* are monadic predicates. The second definition is our text (D).

18. The subject of Stoic *cases* or *case-contents* (πῶσις) is difficult. We assume, with Gaskin 1997, 94–101, and Durand 2018, 73–78, that *ptôseis* are contents. For alternative views, see Bronowski 2019, 352–359; Long & Sedley 1987, 200 v.1; Frede 1994, 13–17; see also the discussion in Atherton & Blank 2003, 324–326. Nothing hangs on this question here.

*katêgorêma* and an event predicate... as «is walking» yields for example «Socrates is walking».<sup>19</sup> (Ammon. *Int.* 44.23–25)

(C) That which is predicated, then, is predicated of an upright noun or [upright] case-content ... If [what is predicated of an upright noun or [upright] case-content] produces a complete sentence, they call it *katêgorêma* or event predicate.<sup>20</sup> (Stephanus, *Int.* 11.9–12)

(These two texts mix Peripatetic with Stoic terminology; cf. fns. 54, 56. A canonically Stoic formulation would not have 'upright noun' and would have 'proposition' or 'complete content' for 'complete sentence'.) The definition of 'katêgorêma' as monadic predicate is thus well-attested.

A second Stoic definition of 'katêgorêma' allows for two readings. First, a reading as an account of monadic predicates. Second, a reading as an account that includes monadic and polyadic predicates:

(D) A *katêgorêma* is ... an object<sup>21</sup> that can be connected with some thing or some things, as Apollodorus<sup>22</sup> says. (DL 7.64)<sup>23</sup>

19. ἂν μὲν οὖν ὀνόματός τι κατηγορηθὲν ἀπόφανσιν ποιῆ, κατηγορήμα καὶ σύμβαμα παρ' αὐτοῖς ὀνομάζεται ... ὡς τὸ περιπατεῖ, οἷον Σωκράτης περιπατεῖ (Ammon. *Int.* 44.23–25). For the Stoic event predicates (σύμβαμα), see below §3. (Square brackets in a translation indicate a phrase supplied by context. Angled brackets in text and translation are used to indicate a textual emendation.)

20. τὸ κατηγορούμενόν τινος ἢ ὀνόματος κατηγορεῖται ἡγουν εὐθείας ἢ πῶσεως. ... καὶ εἰ μὲν αὐτοτελῆ τὸν λόγον ἀπεργάζεται, καλοῦσιν αὐτὸν κατηγορήμα ἢ σύμβαμα (Stephanus, *Int.* 11.9–12).

21. In their logic, the Stoics appear to use *lekton* ('sayable') and *pragma* (which we translate as 'object') synonymously, e.g. DL 7.57, 63. See also Bronowski 2019, 118–119; Atherton 1993, 250; Barnes et al. 1999, 197–198.

22. Reference is to the second-century BCE Stoic Apollodorus of Seleucia, student of Diogenes of Babylon.

23. Ἔστι δὲ τὸ κατηγορήμα ... πρᾶγμα συντακτὸν περὶ τινος ἢ τινῶν, ὡς οἱ περὶ Ἀπολλόδωρον φασιν (DL 7.64). The ancient Greek phrase ὡς οἱ περὶ X φασιν

On *the first reading*, the phrase ‘some thing or some things’ refers to a singular or plural content of which the predicate is predicated (‘*this one walks*’, ‘*these walk*’).<sup>24</sup> This reading is compatible with the definition in texts (A), (B) and (C), which says nothing about the number (singular or plural) of the case-content.<sup>25</sup> It makes the definition of ‘katêgorêma’ as one of monadic predicate, except that it takes pluralities in addition to singularities in subject place (e.g. ‘these’ or ‘some’). Strong independent evidence supports this reading. Thus text

(E) Moreover, they also make this distinction: desire is for those things which are said of some person or several persons, which the [Stoic] logicians call κατηγορήματα, for example «to have riches» or «to receive honours». (Cic. *Tusc. Disp.* 4.21)

contains implicitly a very similar Stoic definition, namely:

(1) A katêgorêma is that which is said of one or more people (or things).<sup>27</sup>

The examples in (E) are analogues to monadic predicates that can be said of one or more persons: ‘someone has riches’, ‘some have riches’.

is almost always just another way of saying ‘as X says’, and we translate accordingly.

24. Long & Sedley 1987, v.2, 199, and Barnes 1999, 204, note the possibility of this interpretation without endorsing it. Hicks 1925; Hülser 1987–88, 809, 933; and Mensch 2018 adopt this reading.

25. Pace Barnes 1999, 204, who assumes that the definition in (A) presupposes the addition of a *singular* nominative case.

26. *Distinguunt illud etiam, ut libido sit earum rerum, quae dicuntur, de quodam aut quibusdam, quae κατηγορήματα dialectici appellant, ut habere divitias, capere honores.* The *de quodam aut quibusdam* could also be translated as neuter (Graver 2002, 46): ‘desire is for those things which are said of some *thing* or some *things*’. This produces a more general definition of monadic predicates. We note that Cicero is generally a reliable source for early Stoic doctrine.

27. We can imagine the Greek: κατηγορήμα τὸ κατὰ τινος ἢ τινῶν ἀγορευόμενον. Cicero’s *quodam aut quibusdam* clearly refers to the upright case-content, the upright πῶσις. (1) appears to be a fusion of the second definition of katêgorêma at DL 7.64 with the first (given in fn. 17 above).

So (E) confirms the first reading. Further corroboration may be the fact that Chrysippus had an interest in singular and plural expressions (DL 7.192) and possibly considered singular and plural katêgorêmata in *Log. Inv.* cols. I.5–7, I.15–20, II.21–26.<sup>28</sup> Singular and plural expressions as possible arguments for Stoic monadic predicates are also mentioned in a later text.<sup>29</sup>

On *the second reading* of (D), the definition would refer to what in contemporary logic would be the arguments the predicate takes (‘*this one walks*’, ‘*Dio sees Plato converse with Socrates*’). The definition would then cover both monadic and polyadic predicates.<sup>30</sup> This reading is sometimes thought to be supported by the second of two Stoic definitions of ‘verb’ (*rhêma*), which displays salient parallels to (D).<sup>31</sup>

(F) A verb is ... , or as some say, a caseless element of speech that signifies something that can be connected with some thing or some things, such as write, speak.<sup>32</sup>  
(DL 7.58)

This definition does not mention katêgorêmata. The parallel to (D) suggests that the signified ‘something’ is a katêgorêma in the sense of that text. Now, a verb may leave room for more than one case-content, and the Stoics were aware of this. So, this definition of ‘verb’ may favour the reading of (D) as a definition of katêgorêma as one that covers monadic and polyadic cases.

28. All pointed out by Barnes 1999, 204 n. 167. Cf. also Frede 1974, 53.

29. ‘It rues *this one*, it rues *these*’ in Ammonius’ report of the Stoic *parasumbamata* (Ammon. *Int.* 44.32, for which see below §§3 and 4.1).

30. This reading is adopted by Gaskin 1997, 93.

31. Noted by Barnes 1999, 203, fn. 65; Atherton 1993, 45–46.

32. ῥήμα δέ ἐστι ... ἢ, ὡς τινες, στοιχείον λόγου ἄπτωτον, σημαίνόν τι συντακτὸν περὶ τινος ἢ τινῶν, οἷον Γράφω, Λέγω. (DL 7.58). ἄπτωτον is often translated as ‘undclinable’ or the like. As Stoic case-contents (πτώσεις) are not linguistic items, the meaning is more likely that verbs do not *signify* case-contents.

We see two possibilities.<sup>33</sup> Either all Stoics used the term ‘katêgorêma’ throughout for monadic predicates only. Or the term ‘katêgorêma’ was originally, and by Chrysippus among others (see §3), used for monadic predicates only, but a couple of generations later, the Stoic Apollodorus added an alternative notion of katêgorêma as that which is signified by a verb and that hence includes both what we call monadic and what we call polyadic predicates. This alternative notion would have been motivated by considerations of *grammar* and, from a *logical* perspective, would be a serious step back compared to the earlier notion, since it no longer provides for one-place predicates of complex forms like  $Fxa \wedge p$  (see §3). Either way, it is certain that Chrysippus and other early Stoics had a *logical* notion of monadic predicates and used ‘katêgorêma’ to express that notion, and moreover that this notion was the prevalent Stoic one ((A), (B), (C), (E) above, and §3). Hereafter ‘monadic predicate’ translates the Stoic ‘katêgorêma’ as and when it is used for that notion.<sup>34</sup>

33. Ultimately, the evidence is not conclusive for either, as Barnes 1999, 204, also concludes.

34. Gaskin 1997 argues that ‘katêgorêma’ was used in a broad and in a narrow way: in the narrow sense, only a verb that lacks nothing but a nominative case-content for completion is a katêgorêma; in the broad sense, any verb signifies a katêgorêma (93, 103). We believe that Gaskin’s view rests on two errors and some unproven conjectures. His main error is in his argument that DL 7.64 (our text (A)) implies that the Stoics insisted that every proposition requires a nominative case, i.e. that it is a necessary condition for something to be a proposition that it contain a nominative case (91–92 and then repeated *passim*). However, DL.7.64 is a definition not of propositions (*axiômata*) but of katêgorêmata. It only commits one to the view that the Stoics maintained that it is a necessary and sufficient condition for something to be a katêgorêma, that completion with a nominative case generates a proposition. Gaskin’s mistaken assumption guides his entire argument, and without it the argument collapses. We believe that Gaskin is also in error when assuming that what is generally accepted to be a lacuna in DL 7.64 contained a *fourfold* distinction (for which see our §4) and that this is usually assumed (92 with fn. 3). The fourfold distinction is not reported for Chrysippus; the twofold distinction between *sumbamata* and *parasumbamata* is; and it is *parasumbamata* alone that are usually assumed to have gone missing in the lacuna. So Gaskin cannot use his conjecture of a *fourfold* distinction in the lacuna to back up his –mistaken– assumption that a nominative case is necessary for something to be a Stoic proposition. Gaskin’s incorrect assumption leads him to

### 3. Monadic predicates (katêgorêmata) as functions

It has been suggested that one can think of Stoic katêgorêmata as functions roughly in the sense in which Frege considered predicates as functions.<sup>35</sup> We agree with this. In fact, we argue that the similarities go further than has been pointed out. There is no direct evidence that the Stoics had a term for ‘function’. Rather, one needs to show that the role katêgorêmata (and related notions) play in Stoic logic provides sufficient evidence for this suggestion. Good initial evidence can be found in the Stoic definitions of three kinds of simple affirmative propositions. Of these there survive two sets. They appear to match in content, and partially in terminology.<sup>36</sup> A ‘definite’ proposition consists of a *demonstrative* upright case-content (*ptôsis*) and a katêgorêma. A ‘middle’ proposition consists of an upright case-content and a katêgorêma. An ‘indefinite’ proposition consists of one or more indefinite parts and a katêgorêma.<sup>37</sup> Examples for definites are «this one is walking», «this one is sitting»; for middle ones «Dio is walking», «Socrates is walking»; and for indefinites «someone is walking», «someone is sitting».<sup>38</sup> The set of definitions and the matching

some further implausible conjectures, such as that the Stoics believed that an upright case-content would, in the *parakatêgorêmata* or *parasumbamata*, be expressed by an *oblique* linguistic expression (95, 98). Gaskin’s view also cannot account for the more complex monadic predicates (*katêgorêmata*) attested for Chrysippus (PHerc 307; see our §3). For our view on the relations between *sumbamata*, *parasumbamata* and *katêgorêmata*, see below fn.54.

35. Frege 1891, 1904. See Gaskin 1997, 94–101; Hülser et al. 2009, 378. Cf. also Egli 1986. Atherton questions the analogy (1993, 45–46, n. 10).

36. Sextus has ‘indefinite’ (*aoriston*), ‘middle’ (*meson*) and ‘definite’ (*hôrismenon*) (M 8.97–100); Diogenes ‘indefinite’ (*aoriston*), ‘categorical’ (*katêgorikon*) and ‘categorical’ (*katêgoreutikon*). (DL 7.70). We follow Sextus’ usage, in part because of the obscurity of Diogenes’ terms. Our focus will be the indefinite propositions, whose terminology is agreed on by both sources. For general discussion of this classification, see Frede 1974, 53–67; Durand 2019, §§2–6; Barnes et al. 1999, 97–100; Brunschwig 1994; Crivelli 1994a; and Ebert 1993.

37. There is a textual problem in Diogenes’ definition of the indefinite, for which see below §5.

38. DL 7.70, SE M 8.96–98. *Anthrôpos* could express the generic, ‘human being’, or what is expressed in English by an indefinite article and a noun phrase, ‘a

examples of the three kinds of affirmative simple propositions suggest that the *katêgorêma* (monadic predicate) is like a function that can be completed by slotting into its argument place an argument of one of three kinds.<sup>39</sup> These show some similarity, in order, to a demonstrative, an individual constant and something like a variable bound by an existential quantifier. The kind of argument that fills the argument place of the *katêgorêma* determines the kind of proposition one gets.

Further evidence is provided by the Stoic notions of complete and incomplete contents. We are told that

(G) Incomplete is a content that has the expression unfinished, for example «...is writing»<sup>40</sup>, for we ask ‘who?’. Complete is a content that has the expression finished, for example «Socrates is writing». *Katêgorêmata* are then among the incomplete contents, and propositions among the complete contents.<sup>41</sup> (DL 7.63)

The incomplete content «is writing» can be completed variously by «Dio», «a human being», «this one», «someone» or «the teacher Kallias»,

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human being’. The context makes clear that here it is the latter. See Barnes et al. 1999, 98. The co-classification of proper names and generic nouns as one class of expressions may seem odd. It is motivated by Stoic metaphysics. See Long & Sedley 1987, 182, v.1; Bailey 2014, 285–290. The reasons why «a human being is sitting» is middle and not indefinite, and is lumped together with «Dio is walking», are again metaphysical rather than logical, and we disregard them, since we are after multiple generality. For details, see e.g. Bronowski 2019, 304–340; Durand 2018, 66 n.6, 169; Bailey 2014, 295–298; and Caston 1999, 187–192.

39. Another example (SE M 8.308): ‘If *some god* tells you that this one will be rich, this one will be rich; but *this god* here (I point at Zeus, by hypothesis) tells you that this one will be rich.’ See further discussion in Durand 2019, §§46–47.
40. As Gaskin 1997, 102–103, points out, the Greek γράφει is ambiguous and leaves unclear whether one should be translated “is writing” or “he is writing”. In any event, it is clear that the content expressed is an incomplete content, i.e. the *katêgorêma*, «...is writing». Cf. Barnes et al. 1999, 203.
41. ἑλλιπῆ μὲν οὖν ἐστὶ τὰ ἀναπάρτιστον ἔχοντα τὴν ἐκφορὰν, οἷον Γράφει· ἐπιζητούμεν γάρ, τίς; αὐτοτελῆ δ’ ἐστὶ τὰ ἀπληρισμένην ἔχοντα τὴν ἐκφορὰν, οἷον Γράφει Σωκράτης. ἐν μὲν οὖν τοῖς ἑλλιπέσι λεκτοῖς τέτακται τὰ κατηγορήματα, ἐν δὲ τοῖς αὐτοτέλει τὰ ἀξιώματα.

just as a Fregean predicate function can be completed by its argument (including, in that case, a variable-binding quantifier).<sup>42</sup>

As to the *value* of a completed predicate function, in the case of propositions, it seems to be a truth-evaluable complete content, or, in other words, the proposition itself: it is only (and precisely) assertible *complete* contents that have a truth-value, either being true or being false. By completing a *katêgorêma* with one of the options for arguments from above, a proposition, and that *is* a content with a truth-value, is generated (cf. text (A)). And one definition of Stoic propositions is that they are complete contents that are either true or false (SE M 8.73; DL 7.65).

Thus there are good reasons to assume that the Stoics themselves thought of a *katêgorêma* as something similar to a function: something that, when completed by an argument, produces a truth-evaluable complete content.<sup>43</sup> This ‘modern’ understanding of *katêgorêma* as function is further confirmed by examples of *katêgorêmata* whose logical structure is of greater complexity. In §2 we argued that the Stoic term ‘*katêgorêma*’ is best translated as *monadic predicate*. The simplest examples were expressed by finite verbal phrases, such as “is walking”, “is writing”, “is thinking” (DL 7.63, 64). A second Stoic definition of ‘verb’ as ‘a part of speech that signifies an uncompounded (*asuntheton*) monadic predicate’ (DL 7.58)<sup>44</sup> suggests that the Stoics have an

42. Since, in Stoic sources, the arguments themselves («Dio», «a human being», etc.) are never called, or adduced as examples of, *incomplete* contents, we assume that they are not incomplete.
43. In some cases in which a *katêgorêma* is completed by an argument, the resulting value is a complete content that is not truth-evaluable, or at least not in any straightforward manner. Examples are a promise or a command. See Chrysippus, *Log. Inv.* col. XIII; DL 7.66–68; and SE M 8.70–74. We do not discuss such non-propositional complete contents here. See Barnes 1986 for discussion of the logic of commands.
44. ῥῆμα δὲ ἐστὶ μέρος λόγου σημαῖνον ἀσύνθετον κατηγορήμα, ὡς ὁ Διογένης (DL 7.58). The definition is attributed to Diogenes of Babylon, pupil of Chrysippus, logic teacher of Carneades.

expression for such basic monadic predicates: *uncompounded* monadic predicates.<sup>45</sup> Their general form would be

(2) ...F or, in contemporary terms,  $Fx$

There are also a considerable number of examples of monadic predicates, explicitly called *katêgorêmata*, that consist of more than what is signified by a verb. First, there are several which have two components: “to sail through rocks”, “to have riches” and “to obtain honours”, and “to drink absinthe”, all expressed with the verb in the infinitive.<sup>46</sup> Just as in the case of uncompounded monadic predicates, their form can be symbolized as

(2) ...F or, in contemporary terms,  $Fx$

though F itself is signified by a complex expression that combines a verb with a generic singular or plural noun, either declined or as a prepositional phrase. This is in accordance with the fact that ‘*katêgorêma*’ is a logical expression as compared with a grammatical one. It is its incompleteness, not its corresponding to what is expressed by a verb, that determines its character. But we can do better.

Chrysippus is our best source for monadic-predicate analogues to contemporary logic. One crucial bit of evidence is from his *Logical Investigations*. Here we find, in consecutive lines, the following

45. *Asuntheta katêgorêmata*: For this expression, see e.g. Mich. Sync. 79: ‘In general, every verb is said to be a *katêgorêma* (i.e. by the Stoics); specifically, there are compounded *katêgorêmata* and uncompounded *katêgorêmata*. A compounded [*katêgorêma*] is one which is combined with the case of a name or a pronoun, whereas an uncompounded [*katêgorêma*] is the verb itself said on its own. The Stoics call the combining compounding.’ (πᾶν ῥῆμα γενικῶς λέγεται κατηγορημα· ειδικῶς δὲ σύνθετον κατηγορημα καὶ ἀσύνθετον κατηγορημα· σύνθετον μὲν ἐστὶ τὸ συντεταγμένον πῶσει ὀνόματος ἢ ἀντωνυμίας· ἀσύνθετον δὲ αὐτὸ τὸ ῥῆμα τὸ καθ’ ἑαυτὸ λεγόμενον· σύνθεσιν δὲ οἱ Στωϊκοὶ τὴν σύνταξιν λέγουσιν.) Note that this late source, as is quite common (see §1), confuses the levels of language and content and does not distinguish between verb and *katêgorêma*. This kind of Stoic use of *asunthetos* in DL 7.58 is also confirmed at SE M 8.136, ‘no proposition is uncompounded’ (οὐδὲν ἀξιωμα ἀσύνθετον). See also Gaskin 1997, 93, for the term ‘uncompounded’ used of *katêgorêmata*.

46. DL 7.64: διὰ πέτρας πλεῖν; cf. SE M 8.297. *Tusc. Disp.* 4.21: *habere divitias, capere honores*; SE PH 2.230, 232: τὸ ἀψίνθιον πιεῖν.

juxtaposition of what is explicitly referred to as a proposition (*axiôma*) and what is explicitly referred to as a monadic predicate (*katêgorêma*) (Col.XIII, 17–22):

(3) The proposition «Dio is walking, but if not, he is sitting.» (Col. XIII lines 17–19)

(4) The *katêgorêma* «to walk, but if not, to sit» (Col. XIII lines 19–22)

In the vicinity, there are similar constructions, which must also be *katêgorêmata* (Col.XI). Of particular interest is

(5) «to walk, since it is day» (Col. XI lines 24–25)<sup>47</sup>

It is the peculiar technical formulation with infinitives which indicates that what is at issue is a predicate (qua incomplete sayable, not qua part of a – complete – proposition).<sup>48</sup> (4) and (5) are of critical importance, since they show that for Chrysippus a monadic predicate is *not* that which is signified by a verb (*rhêma*). (Those are the *uncompounded* monadic predicates.) Rather, a monadic predicate can be signified by a rather complex expression and contain both a plurality of *katêgorêmata* and two-place connectives and a negator (4) and even propositions (5). (Reader, let this sink in, please, because it is extraordinary!) In contemporary symbolism, the analogous monadic predicate to (4) would be perhaps

(6)  $Fx \wedge (\neg Fx \rightarrow Gx)$

and to (5)

47. The Greek is (3) περιπατεῖ Δίων, εἰ δὲ μὴ, κάθηται; (4) περιπατεῖν, εἰ δὲ μὴ, καθῆσθαι; and (5) περιπατεῖν, ἐπεὶ δ’ ἡμέρα ἐ[στι]ν. The cases in Col. XI are referred to without a specific noun. We assume that τὸ ὅλον τοῦτο (Col. XI. 23) is short for τὸ ὅλον τοῦτο κατηγορημα.

48. Cf. the examples in fn. 46 above. See also Barnes 1999, 203 n. 164, who assembles yet more examples (SE PH 3.14, M 9.211; Seneca *Epist.* 117.3, 12; and, for Zeno, Stobaeus *Ecl.* 1.13.1c), and Inwood 1985, 65: ‘Stealing and not stealing are predicates and are indicated in the Greek by the infinitive form of the verb, which is often used to stand for predicates.’

$$(7) p \wedge (p \rightarrow Fx)^{49}$$

It does not matter what contemporary symbolism exactly would capture these two predicates.<sup>50</sup> It matters that, either time, we have as monadic predicate a logical item that, until Frege, would generally not have been called a predicate.<sup>51</sup> How do we know that there are to be three (or two)  $x$ 's rather than an  $x$  and a  $y$  (in contemporary speak), in which latter case we would have a dyadic predicate? First, we know this because we are given (3) as the corresponding proposition (complete content) for (4), with the analogous contemporary form

$$(8) Fa \wedge (\neg Fa \rightarrow Ga).$$

Second, a *katêgorêma* is, by its definition, a monadic predicate (§2). So there is undoubtable evidence that the Stoics (and in particular Chrysippus) have monadic predicates that may contain as components connectives, more than one predicate and whole propositions.

The Stoics made progress over their predecessors with regard to the understanding of predicates as 'objects of logic' also in a further respect. This is their distinction between event predicates (*sumbamata*) (DL 7.64) and secondary-event predicates (*parasumbamata*). The distinction is Chrysippean,<sup>52</sup> and we know little about its origin. Later ancient texts unquestionably understand both Stoic notions as analogues to monadic predicates. They associate event predicates with

49. The corresponding complete content is a quasi-conditional (*parasunêmmenon*). DL 7.71 provides its (Fregean-sounding) truth-conditions. See Barnes et al. 1999, 108–109.

50. Alternatively, one could use Church's lambda calculus. A lambda abstractor would then bind the variables in (4) and (5), and quantification would be applied to the resulting expression. In this way, quantification itself does not bind variables, a situation that arguably comes closer to Stoic quantification than Frege-style quantification does.

51. Such complex monadic predicates also seem to feature in the Stoic theory of action, on which actions (*hormai* in Stoic terms) are directed at *katêgorêmata* (Arius in Stobaeus *Ecl.* 2.9b = 88 Wachsmuth; cf. Cic. *Tusc. Disp.* 4.21, quoted above, and discussion in Inwood 1985, 118–126).

52. Cf. Lucian, *Vit. Auct.* 21. Chrysippus also wrote a work on *sumbamata* — if von Arnim's emendation is correct (DL 7.191). See also Barnes 1996.

monadic predicates ((B) and (C)) and secondary-event predicates with monadic secondary predicates (*parakatêgorêmata*). They define the secondary-event predicate as that which 'yields an assertible content when predicated of an [oblique] case-content (*ptôsis*)' and state that 'it is like a secondary predicate, as in the case of «to ... is regret», for example «to Socrates is regret»' (Ammon. *Int.* 44.25–27).<sup>53</sup> Now, the Stoics find themselves in a long philosophical tradition that takes it as given that the subject-predicate distinction provides the basic components of sentences, with the subject expression generally in the nominative. By introducing the *secondary-event predicates* as a kind of monadic predicate and labelling it 'secondary', the Stoics *increase the scope of what is logically treatable*. In logic, it does not matter whether a sentence that expresses a proposition has a noun (or noun clause) in the *nominative*. What matters is that the sentence reflects the structure of the predicate function, here «to ... is regret», and how it can be completed into a proposition, here «to Socrates is regret». That the argument of the function is an oblique case-content and is expressed with the dative becomes less important for its being a predicate.<sup>54</sup> (Be-

53. ἄν δὲ [πλαγίου] πτώσεως [τι κατηγορηθὲν ἀπόφανσιν ποιῆ], παρασύμβαμα [παρ' αὐτοῖς ὀνομάζεται] ... καὶ ὄν οἷον παρακατηγόρημα, ὡς ἔχει τὸ «μεταμέλει», οἷον «Σωκράτει μεταμέλει» (Ammon. *Int.* 44.25–27). Barnes' translation 'it rues Socrates' is more elegant. Our clumsy rendering is designed to make explicit the relation with the oblique case-content, here expressed by a dative. Bronowski 2019, 425–426, maintains that the Stoics did not consider the combination of *parakatêgorêmata* (and implied less-than-*parakatêgorêmata*) with oblique case-contents as propositions (*axiômata*). Her reasons seem to be that Diogenes Laertius does not mention *parakatêgorêmata* and that they do not occur in Stoic arguments in our surviving evidence. These seem to us to be insufficient reasons. No source directly supports Bronowski's claim that propositions cannot be constituted from *parakatêgorêmata* and oblique case-contents. On the contrary, Ammonius refers to the combination of any species of predicate (κατηγόρημα, παρακατηγόρημα, ἔλαττον ἢ κατηγόρημα and ἔλαττον ἢ παρασύμβαμα) with one or more cases as 'that which we assert' (*apophansis*) (Ammon. *Int.* 44.19–45.6); cf. the Stoic definition of the proposition (*axiôma*) as that saying which we assert (*apophainesthai*) (DL 7.66).

54. DL 7.64 seems to report that some *katêgorêmata* are *sumbamata* and some are *parasumbamata* (if we follow the most common emendation of the text). Later sources associate *sumbamata* with *katêgorêmata*, and for *parasumbamata* we find the new term '*parakatêgorêma*' (see §4.1 for some of those texts).

low in §7 and §8 we will see that the distinction between *sumbama* and *parasumbama* is relevant to the structure of Stoic propositions: in its absence, propositions would be ambiguous, which is incompatible with Stoic theory.)

#### 4. Stoic polyadic predicates

##### 4.1

We have argued that the Stoic *katêgorêmata* are best understood as the Stoic analogues to monadic predicates, and we have noted how advanced this Stoic notion is as a logical notion, from the perspective of contemporary first-order logic. It is generally assumed that the lack of a solution to the problem of multiple generality in traditional Aristotelian logic is in part due to the absence of polyadic predicates. So, the obvious next question is whether the Stoics had dyadic and generally polyadic predicates. In fact, for the nontrivial manifestation of multiple generality, only dyadic predicates are *necessary*. And in fact, they had. It is amply documented that the Stoics distinguished between

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*Parasumbamata* in those later sources are completed with an *oblique* case-content. The third definition of *katêgorêma* (text (D), DL.7.64) entails that *katêgorêmata* require an upright case-content to generate a proposition. So are aren't both *sumbamata* and *parasumbamata katêgorêmata*? We offer two alternative answers. (1) For Chrysippus and some of his successors, both count as *katêgorêmata*. At some later point, the associations just mentioned are introduced, and *parasumbamata* are no longer *katêgorêmata*, but are *parakatêgorêmata*. The definitions in DL 7.64 are assumed to be in chronological order (Atherton 1993, 253 fn. 31). The first two definitions allow for *parasumbamata* as a subclass of *katêgorêmata*. The third one reflects the later associations, in which the notion of *katêgorêma* has narrowed. (2) The Stoics distinguished between a generic and a specific sense of monadic predicate (*katêgorêma*): in the specific sense, it does not include *parasumbamata* or *parakatêgorêmata*; in the generic sense, it does. This distinction differs substantially from Gaskin's, above fn. 34. (It is quite possible that the content of the expressions '*sumbama*' and '*parasumbama*' changed somewhat over time, and started out related to the causative aspect of *katêgorêmata*, but we disregard this point here, since evidence is too scarce for reasonable conjecture. Generally, the status of the pair of expressions *sumbama/parasumbama* and their relationship to the pair *katêgorêma/parakatêgorêma* is not at all clear-cut, and several of the sources seem confused. However, the specific historical and textual difficulties are not relevant to our topic, i.e. multiple generality.)

*katêgorêmata* and certain incomplete contents that they referred to as *less-than-katêgorêmata*.<sup>55</sup>

(H) And again, if what is predicated of a noun requires the addition of a case of a noun to produce an assertion, it is called (or 'said to be') less than a *katêgorêma*, as in the case of «likes» and «favours», for example «Plato likes». For only if *whom* is added to this, for example «Dio», does it produce as a definite assertion «Plato likes Dio». <sup>56</sup> Ammon. *Int.* 44.32–45.3.<sup>57</sup>

The contrast is between *katêgorêmata* ((B) above) and *less-than-katêgorêmata*. A *katêgorêma* plus a noun (or rather a case-content

55. *Less-than-katêgorêmata* are also mentioned in Stephanus, *Int.* 11.18–21; Apoll. Dysc., *On Syntax* 3.402–403; Stobaeus *Ecl.* 2.76; and Scholia to Lucian, *Vit. Auct.* (27) 21.56–60. These texts are of different levels of reliability (see fn. 57). Other than Stobaeus, all contain later ancient Peripatetic or Platonist terminology and conflate the levels of signifiers and signified.
56. καὶ πάλιν ἂν μὲν τὸ τοῦ ὀνόματος κατηγορούμενον δέηται προσθήκης πτώσεως ὀνοματὸς τινος πρὸς τὸ ποιῆσαι ἀπόφανσιν, ἔλαττον ἢ κατηγορήματα λέγεται, ὡς ἔχει τὸ «φιλεῖ» καὶ τὸ «εὐνοεῖ», οἷον «Πλάτων φιλεῖ» τούτῳ γὰρ προστεθὲν τὸ τίνα, οἷον «Δίωνα», ποιεῖ ὠρισμένην ἀπόφανσιν τὴν «Πλάτων Δίωνα φιλεῖ». We here translate '*philei*' with 'likes' rather than 'loves' to bring out more clearly that in ancient Greek there would be an expectation of the object of being liked or loved to be provided.
57. The Ammonius passage (from which (H) and (B) above are excerpted) explicitly says that it reports Stoic theory, and indicates the beginning and end of that report. It uses Stoic terminology and canonical Stoic examples. It is not a fragment taken from an early Stoic text, since it intersperses comparison with Peripatetic theory and repeatedly conflates the Stoic distinction between content and linguistic expression. On the passage and its reliability, see also Barnes 1999, 205; Gaskin 1997, 95; and Long & Sedley 1987, v.2 203–204. There are six surviving parallels to this passage: Stephanus, *Int.* 11.2–21; Anon. *Int.* 3.6–17; Apoll.Dysc., *On Syntax* 3.402–403; Apoll.Dysc., *On Syntax* 3.429; Apoll.Dysc., *On Pronouns* 115.9–13; and Scholia to Lucian, *Vit. Auct.* (27) 21.56–60. Cf. also Priscian, *Inst. Gram.* 18.4; and Suda s.v. *sumbama*. Most are less reliable than Ammonius. The distinction between *katêgorêmata* and *less-than-katêgorêmata* is recorded more or less accurately in all of these: see esp. Stephanus, *Int.* 11.2–21, Apoll.Dysc., *On Syntax* 3.155, and the Scholia to Lucian, *Vit. Auct.* See also Gaskin 1997, 106. Bronowski 2019, 429, assumes without argument that *less-than-katêgorêmata* are *katêgorêmata*, something we do not find plausible.

expressed by a noun, DL 7.64, 70, §2) yields a complete content. An incomplete content that, *in addition to* requiring an upright case-content, requires an oblique case-content to yield a complete content, is not a katêgorêma. It is less than a katêgorêma in the sense that a katêgorêma needs one noun-content for producing a proposition, whereas a less-than-katêgorêma needs (at least) two.

As others noticed, these less-than-katêgorêmata are perfect candidates for Stoic dyadic predicates (Barnes 1999, 205–206). (H) could be a passage in a contemporary logic textbook that explains dyadic predicates: The student may expect «Plato likes» to be a proposition or complete content, but it is not. Just as «... is walking» requires «Socrates» for completion, so «Plato likes...» *still* requires something, for instance «Dio», for completion. «likes» and «favours» have two argument places. Our texts are very clear that this is how the Stoics conceived of them. ‘They call it *less than a* katêgorêma, because it is not a complete katêgorêma.’<sup>58</sup> The Stoic standard form is a place for an upright case-content (...), followed by a place for an oblique case-content (/---), followed by less-than-a-katêgorêma; with F for ‘loves’:

(9) ... /---F

With individual variables and *F* for ‘loves’, this corresponds roughly to the contemporary

(10) *Fxy*

In our most reliable evidence on less-than-katêgorêmata, some (probably early) Stoics applied the notion of less-than-katêgorêmata

58. Cf. e.g. ‘... but «Socrates likes», since the ‘whom’ is missing even though the subject is taken in the upright, [that is] since the proposition is not complete, they call it *less than a* katêgorêma, because it is not a complete katêgorêma.’ (τὴν δὲ Σωκράτης φιλεῖ, ἐπειδὴ λείπει τὸ τίνα, κἂν κατ’ εὐθείαν ἐλήφθη ὁ ὑποκείμενος, ἀλλ’ οὖν, ἐπεὶ μὴ αὐτοτελής ἢ πρότασις, ἔλαττον ἢ κατηγορήμα φασὶν αὐτὴν, ὅτι μὴ τέλειόν ἐστι κατηγορήμα. Scholia to Lucian, *Vit. Auct.* (27) 21.54–58.) The passage and its context strongly suggest that the author confounds propositions and predicates; alternatively something else has gone wrong with the text.

when explaining Zeno’s and Cleanthes’ views of the end or *telos*.<sup>59</sup> Here ‘to live in agreement (with)’ is said to be something less <than a> katêgorêma, which (so the text implies) becomes a katêgorêma if ‘with nature’ (*têi phusei*) is added as second argument, namely ‘to live in agreement with nature’. This application of ‘less than a katêgorêma’ in conjunction with its definition and the descriptions as ‘less than a katêgorêma’ and ‘not a complete katêgorêma’ suggest that the early Stoics conceived of katêgorêmata and less-than-katêgorêmata as ‘nested’ functions. First the (non-subject) argument place in less-than-katêgorêmata is filled (from *Fxy* to *Fxa*). This yields a (complete) katêgorêma (*Fxa*). Then the subject-argument-place is filled (from *Fxa* to *Fba*). This yields a complete content.<sup>60</sup> In our Stoic notation, this is from ... /---F to ... /aF to *b/aF*, filled from inside out.

Consistent with their distinction between event predicates (*sumbamata*) and secondary-event predicates (*parasumbamata*) (§3), the Stoics also defined less-than-secondary-event-predicates. The latter stand to secondary-event predicates as less-than-katêgorêmata stand

59. Arius in Stobaeus *Ecl.* II.7.6, tr. Long & Sedley modified: ‘Zeno rendered the end as: “living in agreement” ... His successors, further articulating this, expressed it thus: “living in agreement with nature”, since they took what Zeno said to be less <than a> predicate. Cleanthes ... added “with nature”, and rendered it thus: “the end is living in agreement with nature”.’ (Τὸ δὲ τέλος ὁ μὲν Ζήνων οὕτως ἀπέδωκε, τὸ ὁμολογουμένως ζῆν ... Οἱ δὲ μετὰ τοῦτον, διαρθροῦντες, οὕτως ἐξέφερον, ὁμολογουμένως τῆ φύσει ζῆν· ὑπολαβόντες ἔλαττον εἶναι <ἢ> κατηγορήμα τὸ ὑπὸ τοῦ Ζήνωνος ῥηθέν. Κλεάνθης γὰρ ... προσέθηκε τῆ φύσει, καὶ οὕτως ἀπέδωκε, τέλος ἐστὶ τὸ ὁμολογουμένως τῆ φύσει ζῆν.) Stobaeus’ excerpts from Arius are a very reliable source for early Stoic theory.

60. Could it not be the other way about, from *Fxy* to *Fay* to *Fab*? Possible but unlikely. The Stoics had a term for the *Fxa* analogue (katêgorêma). They do not offer one for an *Fay* analogue. «... likes Dio» is a (complete) katêgorêma, more precisely an upright (or active) one. There is no analogue term in Stoic theory for «Plato likes...». The *purpose* of passage (H) is to make it clear why it would be wrong to think of «...likes---» as a katêgorêma: it would be wrong since «Plato likes...» is not a proposition; i.e. application of the definition of katêgorêma (as defined in text (A)) fails. This reasoning in (H) says nothing about the order of logical construction. The fact that «...likes---» is ‘not a complete katêgorêma’, (fn. 58) plus the Stobaeus passage (fn. 59), strongly suggests that you go from something that is not a complete katêgorêma to something that is a complete katêgorêma (of the kind we have in §§2 and 3), and from there to a proposition.

to katêgorêmata. A Stoic example is ‘cares for’ (‘to ... for --- there is care’), completed as ‘Socrates cares for Alcibiades’, with **G** for ‘cares for’:<sup>61</sup>

(11) /... /---**G**

Evidently, these are also dyadic predicates. The contemporary analogue, with **G** for ‘cares for’, is

(12)  $Gxy$

We have been unable to find examples in which both argument places are filled with the same argument, e.g. Dio loves Dio (Δίων Δίωνα φιλεῖ), but we keep looking. The Stoic definitions of their two kinds of dyadic predicates are compatible with the same argument filling both argument places. As analogy, we offer that in Stoic definitions of non-simple propositions it is stated that the same proposition can be used twice, e.g.  $p \rightarrow p$  (SE *M* 8.93), and in their theory of indemonstrables, arguments of the form  $p \rightarrow p$ ,  $p \vdash p$  count as first indemonstrables just as those of form  $p \rightarrow q$ ,  $p \vdash q$  (Barnes et al 1999, 136). Since none of these points is compelling, we leave the question open.<sup>62</sup>

#### 4.2

Next we argue that, in the Stoic view, less-than-katêgorêmata could take Stoic quantifying expressions (*tis*, *tis-ekainos*) instead of individual

61. Thus Ammon. *Int.* 45.3–5, tr. Barnes modified: ‘If what is predicated of a case-content (*ptôsis*) needs to be put together with another oblique case-content to make an assertion, it is said to be *less than a secondary-event predicate*: thus “there is care” e.g. “to Socrates for Alcibiades there is care” (i.e. ‘Socrates cares for Alcibiades’). (ἄν δὲ τὸ τῆς πτώσεως κατηγορούμενον ἢ τὸ δεόμενον ἐτέρα συνταχθῆναι πλάγιᾷ πτώσει πρὸς τὸ ποιῆσαι ἀπόφανσιν, ἔλαττον ἢ παρασύμβαμα λέγεται, ὡς ἔχει τὸ μέλει, οἷον Ἰσωκράτει Ἀλκιβιάδου μέλει.) See further discussion in Barnes 1999, 205.

62. Arguably, someone’s killing oneself, cutting one’s own hair, etc. are different kinds of relations from someone’s killing someone else, cutting someone else’s hair (try it!), etc. The general assumption in contemporary first-order logic that one can fill all argument places with the same argument seems natural if, like Frege, one uses mathematical examples when introducing functions with two arguments (Frege 1891, 27–28). It is not essential to the notion of a polyadic predicate.

constants in either argument place. With the definitions of the three kinds of simple affirmative propositions in mind (§3), we expect this.<sup>63</sup> The illustrations of less-than-katêgorêmata contain only proper names. There are several Stoic examples of what are likely to be Stoic polyadic predicates that have indefinite parts in subject place.<sup>64</sup> Perhaps most valuable is this Stoic example of a *plausible proposition*, which will engage us repeatedly:<sup>65</sup>

(**I**) A proposition is plausible if it provokes assent, for example «If someone gave birth to something, then she is its mother».<sup>66</sup> (DL 7.75)

(The sentence that expresses the proposition in (**I**) is what linguists call an if-clause donkey sentence.<sup>67</sup>) In its ‘antecedent’, the proposition in (**I**) gives an illustration of a proposition that contains a (potential) two-place predicate with two indefinite particles as arguments: ‘gave birth to’ with ‘someone’ and ‘something’. In form, this predicate is analogous to

63. The extant definitions of ‘being less than a katêgorêma’ (texts in fns. 56, 58) use non-Stoic terminology for the arguments: *onoma*, *ptôseôs onomatos*, etc., whereas DL 7.70 has the Stoic *orthê ptôsis*, *deiktikê orthê ptôsis* and *aoriston morion*.

64. «If some god tells you that *this one* will be rich, *this one* will be rich» (SE *M* 8.308, Greek in fn. 69) combines indefinite part, case-content and demonstrative case-content. The Chrysippean-based «if someone was born in the sign of Sirius, that one will not die at sea» (Cic. *Fat.* 12–14) arguably contains «...born in the sign of ---» and «...will die (at) ---». «When someone is in Megara, he is not in Athens» might have been thought to contain «...is in ---».

65. A *plausible proposition* (*pithanon axiôma*) is a Stoic proposition that inclines us towards assent, even if false (DL 7.75). Cf. the Chrysippean book title: *On plausible conditionals* (DL 7.190), which suggests examples like the one in text (**I**) may originate with Chrysippus. See also Barnes 1985.

66. DL 7.75: εἰ τις τι ἔτεκεν, ἐκείνη ἐκείνου μήτηρ ἐστὶ.

67. *Donkey sentences* contain a pronoun whose anaphoric reference is intuitively clear but whose syntactic function escapes straightforward linguistic analysis. A standard example is ‘Every man who owns a donkey beats it’; see e.g. Geach 1962. An *if-clause donkey sentence* is a donkey sentence that starts with an if-clause instead of e.g. a universal quantifying expression.

(13)  $Fxy$ 

In Stoic terms, «... to --- gave birth» would be less than a *katêgorêma*. It appears then that, in the Stoic view, less-than-*katêgorêmata* could take Stoic quantifying expressions along with individual constants or demonstratives in either argument place. So the Stoics had the kind of notion of dyadic predicate that is required for multiple generality.

4.3.

Logicians are bound to ask at this point: What about (analogues to) predicates with more than two argument places? Alas, no Stoic source explicitly mentions or discusses such cases.<sup>68</sup> The Stoics could have easily defined three-or-more-place predicates recursively on the basis of their definition of dyadic predicates. For the *term* of a three-place predicate, the phrase ‘less-than-a’ could be prefixed to ‘less-than-a-*katêgorêma*’. Generally, if a (less-than-a)<sup>n</sup>-*katêgorêma* is a polyadic predicate, then a (less-than-a)<sup>n+1</sup>-*katêgorêma* is a polyadic predicate. We could also say that a (less-than-a)<sup>n</sup>-*katêgorêma* is the analogue to an (n+1)-place predicate, with  $n \geq 0$ . This is not that farfetched: the Stoics use iterative definitions in many parts of their logic. Conjunctive propositions are said to consist of two propositions and a conjunctive connective (DL 7.72), and negations are defined as propositions that start with the prefix ‘not:’ (Apul. *Herm.* 191.6–11; SE M 8.103; cf. SE M 8.89; DL 7.73). This accounts for conjunctive and negative propositions of any complexity (Barnes et al. 1999, 105–106). Still, there is no direct proof that the Stoics defined three-or-more-place predicates. What about the following Stoic example?

(J) If *some god* tells you that *this one* will be rich, this one will be rich.<sup>69</sup> (SE M 8.308, cf. PH 2.141, italics ours)

68. Some may think that the second Stoic definition of *katêgorêma* (text (D)) attests a generic use of *katêgorêma* and refers to possible argument places (above §2). Accordingly they may suggest that such polyadic predicates would be covered. We have argued (above §2) that, if understood as early Stoic, (D) should not be read in this way.

69. SE M 8.308: εἴ τις σοι θεῶν εἶπεν ὅτι πλουτήσῃ οὗτος, πλουτήσῃ οὗτος.

Does it not contain a predicate «... tells --- that \*\*\* will be rich», representable as

(14) ... /--- \*\*\*H

and analogous in form to the contemporary

(15)  $Hxyz$  ?

We doubt it: one would expect the Stoics to have treated indirect speech differently (cf. the function of ‘said’ in the Stoic *unmethodical arguments*, e.g. Alex. *An. Pr.* 22.17–26). Clear evidence for Stoic three-or-more-place predicates is thus absent. Of course, for a logic to contain multiple generality and to reflect on its problems, dyadic predicates suffice entirely.

### 5. Variable-free predicate logic: monadic predicate logic

The Stoics have no individual variables. (The question whether they had propositional variables is of no concern here.) *A fortiori*, they have no variable-binding quantifiers. We are looking at a *variable-free predicate logic*. Such a thing is not unknown in contemporary logic. Quine’s short and splendid paper ‘Variables Explained Away’ provides a method that allows one to replace Frege-style individual variables and quantifiers with a set of operators.

The Stoics appear to distinguish between two kinds of indefinite propositions. The first is an analogue to contemporary existential propositions.

(K) According to them (i.e. the Stoics), indefinite are those <propositions> in which an indefinite part (*morion*) governs, such as «someone is sitting».<sup>70</sup> (SE M 8.97)

70. SE M 8.97: ἀόριστα δὲ ἐστὶ κατ’ αὐτοῦς ἐν οἷς ἀόριστόν τι κυριεύει μόριον, οἷον τις κάθηται.

(L) An indefinite [simple proposition] is one that is composed from an indefinite part ... and <a katêgorêma>, such as «someone is walking» ....<sup>71</sup> (DL 7.70)

This first kind of indefinite proposition is composed of one indefinite part<sup>72</sup> and a monadic predicate. The indefinite part governs — that is, it has widest scope — in these propositions. Given the Stoic Scope Principle (S1), ‘someone’ or ‘something’ (*tis, ti*) is thus the characterizing expression in sentences that signify these indefinite propositions. The examples in (K) and (L) are examples of such indefinite simple propositions. With  $\tau$  (from *tis, tinos, etc.*) for the indefinite part, we can represent their (Stoic) form as

(16)  $\tau F$

The contemporary analogue is

(17)  $\exists x Fx$

This is confirmed by the explanation of why indefinite propositions are indefinite

(M) «Someone is walking» is indefinite, since it does not demarcate any one of the particular walking individuals. For it can generally be expressed in the presence of any of the particular walking individuals.<sup>73</sup> (SE M 8.97)

We understand (M) as conveying that indefinite propositions *do not refer*. The reason is that they do not pick out any particular person or

71. DL 7.70: ἀόριστον δέ ἐστι τὸ συνεστὸς ἐξ ἀορίστου μορίου ... καὶ κατηγορήματος, οἷον τις περιπατεῖ... . We accept von Arnim’s addition of <καὶ κατηγορήματος>, an emendation which seems uncontested.

72. Texts (K) and (L) imply that the indefinite *morion* is part of the proposition and hence at the level of content. We translate *morion* as ‘part’ and use ‘particle’ for the corresponding linguistic expression. There is no Stoic equivalent to this distinction; it is merely for clarity.

73. SE M 8.97: τὸ μὲν οὖν «τις περιπατεῖ» ἀόριστόν ἐστιν, ἐπεὶ οὐκ ἀφώρμικέ τινα τῶν ἐπὶ μέρους περιπατούντων· κοινῶς γὰρ ἐφ’ ἐκάστου αὐτῶν ἐκφέρεσθαι δύναται· (‘in the presence of’: LSJ s.v. ἐπί A.1.2.e).

thing, since they can be — truthfully — expressed in the presence of *any* particular walking, sitting, etc. person or thing. It is immaterial which person or thing.<sup>74</sup>

*Non-reference* is confirmed by the Stoic truth-conditions for simple (affirmative) *existential indefinite propositions*. The indefinite proposition «Someone is walking» is said to be true precisely if the corresponding definite proposition («this one is walking» accompanied by an indicating of a specific person) is true (SE M 8.98).<sup>75</sup> (By a *corresponding definite* or *middle proposition* we mean henceforward a proposition that differs from the indefinite proposition at issue only in that it has a case-content or demonstrative case-content in place of an indefinite part.) We can confidently conclude that the indefinite propositions of form (16) are Stoic existential propositions with monadic predicates and one quantifier expression. We call them monadic indefinite *existential* propositions.

The second kind of indefinite proposition is somewhat harder to pin down. Based on what we said about the Stoic notion of monadic predicate, we argue that the full sentence behind (L) also provides an example of a monadic indefinite *universal* proposition. This requires a close look at that full sentence.

(N) An indefinite [simple proposition] is one that is composed from an indefinite part or indefinite parts and <a

74. As noted above, Stoic referring expressions appear to have a ‘case-content’ (*ptôsis*) at the content level. Proper names, demonstratives and nouns (in certain functions) each have a corresponding *ptôsis*. For demonstratives, they are called ‘demonstrative case-contents’. The particles *tis* and *tis-ekinos* have corresponding ‘indefinite parts’ (*morion*) at the content level. No indefinite *ptôsis* or case-content is ever mentioned. Neither ‘*tis*’ nor the anaphoric ‘*ekinos*’ (for which see below) nor the combination of the two are ever called *ptôsis* in Stoic texts. See also Crivelli 1994a, 189.

75. A definite proposition is true if that which is pointed at falls under the predicate (SE M 8.100). The account of the truth-conditions of indefinite propositions leads to well-known problems for certain Stoic propositions, such as «someone is dead» and «this one is dead», which we can here ignore, see for a variety of interpretation Barnes et al. 1999, 98–101; Bailey 2014, 281–284; Durand 2019, §§29–34; Bronowski 2019, 415–418.

katêgorêma», such as «someone is walking», «if someone is walking», he is moving». <sup>76</sup> (DL 7.70, continuation of (L) with gap in (L) filled)

We assume that the second example is of the second kind, that is, an indefinite simple proposition with *more than one* indefinite part and a monadic predicate. We agree with and adopt (in (N)) a suggested emendation that assumes a textual lacuna by — very plausible — haplography. <sup>77</sup> Then we have as second example

(18) «If someone is walking, he is moving.»

It contains the indefinite parts ‘someone’, which is non-anaphoric, and ‘that one’, which is anaphoric with cross-reference to the first indefinite part. <sup>78</sup> The result is, in a well-attested canonical form, what the Stoics

76. DL 7.70: ἀόριστον δέ ἐστι τὸ συνεισὸς ἐξ ἀόριστου μορίου ἢ ἀόριστων μορίων <καὶ κατηγορήματος>, οἷον τις περιπατεῖ, <εἰ τις περιπατεῖ> ἐκεῖνος κινεῖται. More literally, ἐκεῖνος κινεῖται translates as ‘that one is moving’. Since we argue (below) that the regimented sentences that express universal propositions use forms of ἐκεῖνος as *anaphoric pronouns*, the natural translation in English is with a personal pronoun. (Here we follow Crivelli 1994a.) This has the additional advantage that elements of case and gender in the Greek can be rendered directly. In some cases in which it increases clarity, we render ‘that one’.

77. The text without emendation is highly puzzling. Cf. the detailed fn. Frede 1974, 59–60, and Crivelli 1994a, 189: ‘One wonders why both examples offered by Diogenes are simple indefinite propositions consisting of one indefinite particle and one predicate, whilst no example is given of a simple indefinite proposition consisting of more than one indefinite particle and one predicate.’ The emendation was suggested by Egli 1981, in conjunction with an alternative to von Arnim’s emendation (fn. 71 above), which we do not adopt, and which does not change the sense of the definition as given here. Egli’s emendation is also adopted by Hülser 1987–88, 1142 (= FDS 914). Crivelli tries to make sense of the text by stipulating a Stoic distinction between anaphoric and non-anaphoric simple indefinite propositions (1994a, 189). However, no texts attest such a distinction, and we consider the conjecture of anaphoric simple indefinite propositions philosophically awkward.

78. Kneale & Kneale 1962, 146, suggest that *ekeinos* must be anaphoric, cross-referring to an antecedent in a conditional. We do not claim that the Stoics considered occurrences of *ekeinos* without a preceding form of *tis* in a preceding clause as anaphoric. (That would be silly.) Equally do we not consider the fact that, in a work on *psychology* in a strange etymological explanation of ‘*egô*’ utilized to corroborate the Stoic placement of the mind in the heart, Chrysippus

call an indefinite conditional. Of these a good number of examples survive. <sup>79</sup> The *very same* example is also attested in Augustine *Dial.* 3, which is based on Stoic logic. <sup>80</sup> (It speaks in favour of the emendation that it results in a Stoic canonical formulation and a known Stoic example.)

It is often assumed that the indefinite conditionals are Stoic *non-simple* propositions, more precisely, conditionals (*sunêmmena*). <sup>81</sup> (Recall that Stoic propositions are either simple or non-simple, §1.) This is however both implausible and problematic. Nor is it entailed by the name: ‘indefinite’ may function as an alienans adjective. It is *implausible*, since the Stoics, in their language regimentation, require that in a proper conditional the subject expression of the antecedent sentence

implies that *ekeinos* has a demonstrative use (Gal. *Plac. Hipp. Plat.* 2.2.7–12) as detrimental to the claim that in logic the Stoics regimented the use of the pair *tis / ekeinos* in such a way that it counted as a pair of indefinite part(icle)s in which the second part does anaphoric duty: The pair of particles works together as a unit, just like the Stoic pairs of connectives *kai ... kai---* and *êtoi ... ê---*. See also Crivelli 1994a, 196; Caston 1999, 196–197; Durand 2019, §§43–45, 51–54. We note that *ekeinos* seems never to occur in demonstrative function in the many demonstrative examples in Stoic logic.

79. For some of them, see Cic. *Fat.* 15; DL 7.75; SE *M* 11.8–13, three examples, for helpful discussion of which, see Crivelli 1994b, 498. Cf. also Cic. *Acad.* 2.20–21; SE *M* 1.86; Plutarch, *Com. Not.* 1080c. Epict., *Diss.* 2.20.2–3 seems to provide evidence as to the reformulation of a negative universal, ‘No *F* is *G*’. Cf. Crivelli 1994a, n. 36.

80. Cf. the conditionals mentioned at DL 7.78, «If Dio is walking, Dio is moving» (with an uncontested emendation by von Armin); and Gellius 16.8.9 «If Plato is walking, Plato is moving». Consistent with the standard practice in Stoic logic, these conditionals are offered as stock examples and are assumed true. We argue that (N) provides (18) as another stock example of a true conditional, but now one that is indefinite. Simple probability calculations show that this is more likely than that we have «someone is walking» and «that one is moving» as two isolated examples, which just happen to form a familiar stock example when put together. Cf. SE *M* 8.100, where «someone is walking» and «someone is sitting» are given as two isolated examples of indefinite propositions.

81. So Frede 1974, 59 fn. 11; Goulet 1978, 205; Crivelli 1994a, 198–199; Bobzien 1998, 156–159. Exceptions are Durand 2018, 167–169; Frede 1974, 64–67 (not entirely clear); and possibly Long & Sedley 1987, v.1, 207 (not entirely clear either).

be repeated in the consequent sentence, if the subject is the same; e.g. “If Plato lives, then Plato breathes”. The practise (we call it *anaphora removal*) applies to all non-simple propositions and is often, though not uniformly, followed in the sources.<sup>82</sup> It ensures that it is discernible at the sentence level that we have two independent simple propositions that are combined in accordance with the Stoic definition of a conditional into a non-simple one. It becomes thus discernible that the consequent is *detachable* e.g. with a Stoic first indemonstrable, which has the form of *modus ponens*. The relevant non-simple *proper* conditional, composed of two simple indefinite propositions, is this conditional:

(19) If someone is walking, someone is moving.<sup>83</sup>

(19) satisfies the Stoic definition of conditionals as non-simple propositions that are composed with the conditional connective ‘if’ (DL 7.71). By contrast, Stoic indefinite conditionals resist anaphora removal, and neither do they satisfy the definition of the conditional, nor is it possible to detach their ‘consequent’ with *modus ponens*. It would be *absurd* to assume that the Stoics did not see the difference between (18) and (19). And there is evidence that they *did* see it.<sup>84</sup>

The assumption that the indefinite conditionals are non-simple is *problematic*, since the Stoics would be shown to be unaware not just of the difference between the sentences expressing (18) and (19), but

82. Cf. e.g. DL 7.77, 78, 80; SE M 8. 246, 252, 254, 305, 308, 423; SE PH 2. 105, 106, 141; Gellius 16.8.9; Gal. *Inst. Log.* iv.1; *Simp. Phys.* 1300; *Alex. An. Pr.* 345; *Cic. Fat* 12. No surviving ancient source explicitly discusses the Stoic convention of anaphora removal. We believe that the frequency with which anaphora removal occurs, together with the fact that it sounds as unidiomatic and is as rare in ancient Greek as in English, is sufficient evidence. It is likely that *because* it is not idiomatic in some cases in which this regimentation is not followed, these are scribal changes; and that for the same reason in other, especially later, ancient cases, some authors are unaware of the convention and disregard it. See Barnes et al. 1999, 104–105, for discussion of Stoic anaphora removal.

83. Cf. Barnes et al. 1999, 112; also Egli 2000, 20–21.

84. *Cic. Fat.* 15; see Bobzien 1998, 156–159. Egli 2000, 20–21, argues plausibly that the Stoic solution to the Nobody Paradox shows that the Stoics distinguished between a pair of sentences similar to (18) and (19).

more generally between conditionals and universal propositions. (For example, in terms of traditional Fregean-Russellian logic, in (19) the conditional connective has wide scope; in (18) the universally quantifying indefinite parts do.) That they were not unaware of this is clear from the fact that, in the Stoic view,

(20) If something is human, it is a mortal, rational animal.

expresses a universal proposition (*katholikon* [*axiōma*]), in this case a definition, and someone who utters (20) says the same proposition as someone who utters

(21) (Every) human is a mortal rational animal.<sup>85</sup>

In (21), and according to the Stoic Scope Principle, ‘every’ would have largest scope.<sup>86</sup> Given the stated synonymy of (20) and (21), ‘something ... it ...’ must have largest scope in (20). Accordingly, in our Stoic examples, the two-particle quantifying expression always has each part as far to the front of its clause as is possible. For the Stoics, (21) expresses a simple proposition, and with (20) and (21), one says the same proposition. But the same proposition cannot be both

85. SE M 11.8: ‘for the one saying “Man is a mortal rational animal” says the same thing in meaning, though different in expression, as the one saying “if something is a man, it is a mortal rational animal”’ (ὁ γὰρ εἰπὼν “ἄνθρωπος ἐστὶ ζῶον λογικὸν θνητόν” τῷ εἰπόντι “εἴ τί ἐστιν ἄνθρωπος, ἐκεῖνο ζῶον ἐστὶ λογικὸν θνητόν” τῇ μὲν δυνάμει τὸ αὐτὸ λέγει, τῇ δὲ φωνῇ διάφορον). What one says (*legei*) is the proposition (e.g. DL 7.66). Hence those two speakers say the same proposition. The following sentence (SE M 11. 9) leaves no doubt that the singular noun without article ‘man’ is understood universally, as covering every man (i.e. human being). For *katholikon* used for an indefinite conditional, cf. SE M 1.86, *Epict. Diss.* 2.20.2–3, *Plut. Com. Not.* 1080c; see also Crivelli 1994a, fn. 36, and Caston 1999, 195–199, esp. 197.

86. Note the example SE M 11.10–11: ‘For the person who carries out a division in this manner, “among human beings, some are Greeks and some are barbarians [i.e. non-Greek speakers]” says something equal to “if some things are human, they either are Greeks or are barbarians”’ (ὁ γὰρ τρόπῳ τῷδε διαιρούμενος “τῶν ἀνθρώπων οἱ μὲν εἰσιν Ἕλληνες, οἱ δὲ βάρβαροι” ἴσον τι λέγει τῷ “εἰ τινὲς εἰσιν ἄνθρωποι, ἐκεῖνοι ἢ Ἕλληνες εἰσιν ἢ βάρβαροι”). Here the leading expression ‘among human beings’ (τῶν ἀνθρώπων) has widest scope. This also shows that the Stoics consider such propositions, with a generic noun without article as subject expression, as a universal proposition of sorts.

simple and non-simple. Hence (20), and by generalization (18), and Stoic indefinite conditionals generally, express simple propositions. As such, they should consist of indefinite parts and a predicate (DL 7.70, above). This is exactly what they consist of. The indefinite parts ‘something’ and ‘it’ work together *as a unit* (just like the parts ‘either ... or ...’, ‘and ... and ...’ in *non-simple* propositions (§1)). The remainder of the content

(22) «If ... is walking, ... is moving»

is a *monadic* predicate by the definition of monadic predicate (katêgorêma). We have evidence that the Stoics, Chrysippus in particular, had that *sort* of predicate (above (4), (5)). The logical form of the propositions (and the regimented sentences expressing them) can then be represented as

(23) Conditional conjunction + indefinite particle ( $\tau_1$ ) ( + cases) + finite verb + corresponding anaphoric particle ( $\epsilon_1$ ) ( + cases) + finite verb.

We always assume that any Stoic regimentation is (in Greek) grammatically correct. In contemporary symbolism with variables, (22) has the form of a monadic predicate

(24)  $Fx \rightarrow Gx$

and the indefinite proposition (18) has the form

(25)  $\forall x (Fx \rightarrow Gx)$

So we can make sense of a perplexing Stoic definition with examples (in (N)) if (i) we accept a very plausible emendation that establishes a Stoic stock example in Stoic canonical form, and (ii) we then take the restored text *to mean exactly what it says*. This solves the conundrum of Stoic indefinite conditionals: They are not non-simple but simple propositions, just as the text implies. (iii) These are constructed with two indefinite parts that form a logical unit and have widest scope in

the proposition, plus a monadic predicate as defined by the Stoics, for which we have evidence elsewhere.<sup>87</sup>

There are then Stoic indefinite universal propositions to complement the Stoic indefinite existential propositions; both kinds are simple propositions.<sup>88</sup> There is evidence for their semantics, too (SE M 11.8–13).<sup>89</sup> It is based on what are called subordinate instances of universal propositions.<sup>90</sup> These are themselves propositions: They can be false (ibid.), and accordingly, true. The instances are proper conditionals with definite (or middle, see below §6) component propositions, in which the case-content is the same in both component propositions. If there can be a false subordinate instance, the indefinite universal is false. We infer that if there *cannot be* any false subordinate instances, the indefinite universal is true.

Why ‘cannot be any’ and not ‘are no’? We assume that the ‘if’ in the universals, or indefinite conditionals, has the same logical strength as in the Stoic proper conditionals (cf. Cic. *Fat* 11–17; Bobzien 1998, 156–159). This is also suggested by their name. So ‘can be’ is required by the – Chrysippean – truth-conditions for Stoic conditionals, which contain a modal element: a conditional is true when its antecedent and the negation of its consequent are incompatible (DL 7.73). However, Chrysippus could and did also account for a kind of non-simple proposition that corresponds to material conditionals, i.e. to the *Philonian conditional*. He (and some other Stoics, it seems) used negations of conjunctions with the antecedent of a Philonian conditional as first conjunct and the contradictory of the consequent as second conjunct as the correct form of that Philonian conditional. (The Philonian

87. For additional evidence that the Stoics were aware that indefinite conditionals were not non-simple propositions (and as such not the kind of conditionals as are part of propositional logic), see §9.

88. We do not here discuss the question whether the two are duals and interdefinable. For their negations, see Barnes et al. 1999, 113–114; Egli 2000.

89. See also Crivelli 1994a, 193–194, 199–202; Barnes et al. 1999, 113; confirmed by DL 7.75, see below §6.

90. ὑποτασσομένου (SE M 11.11); ὑποταχθέντος (SE M 11.9).

conditionals are thus accurately rendered as truth-functional, since they are really negated conjunctions and Stoic negation and conjunction are both truth-functional.) Moreover, Chrysippus' logic contained corresponding *indefinite* negations of conjunctions, too. We assume, by analogy to Chrysippean indefinite conditionals, that the Chrysippean negated conjunctions that replace Philonian conditionals are true when there *are* no false subordinate instances. (Cf. Cic. *Fat* 11–17 and Bobzien 1998, 156–159, on this point.) From a contemporary perspective, the Stoics have two kinds of universal propositions: one containing a modal element, the other being non-modal and truth-functional. For differentiation, we call them the *Chrysippean universal* and the '*Philonian*' universal. (No such names are known from antiquity.) We generally consider Chrysippean conditionals and Chrysippean indefinite conditionals as paradigm. The case for negated conjunctions is analogous, and we mention them only occasionally.

Virtually all extant examples of Stoic universals are *restricted* universals in the sense that universal quantifiers have only conditionals or negated conjunctions in their immediate scope: Not 'everyone is moving', but 'if someone is walking, that one is moving' or 'not: both someone is walking and not: that one is moving'. The closest to non-restricted universals is 'if some things exist, those things either are good or are bad or are indifferent'.<sup>91</sup> (For the Stoics, 'existence' (*einai*) is reserved to bodies, and so is narrower than the ontological category 'something' (*ti*), which includes both bodies and non-bodies.) Still, the example suffices to illustrate by analogy how *unrestricted* universals could be expressed within the Stoic framework: 'if something is something, then that thing ...' (*ei ti ti estin, ekeinon...*). If so, all Chrysippean universals could be expressed canonically in conditional form. Correspondingly, all 'Philonian' universals could be expressed canonically in the form of negated conjunctions (Cic. *Fat* 15–16). Very roughly, the first would correspond to the contemporary  $\forall x \square (Fx \supset Gx)$ ,<sup>92</sup> where

91. SE *M* 11.11: εἰ τινὰ ἐστὶν ὄντα, ἐκεῖνα ἤτοι ἀγαθὰ ἐστὶν ἢ κακὰ ἐστὶν ἢ ἀδιάφορα.

92. Why not  $\square \forall x (Fx \supset Gx)$ ? (i) The surviving argument form does not require it

' $\square$ ' indicates whatever modal force Chrysippean indefinite universal conditionals have, and the second to  $\forall x (Fx \supset Gx)$  or  $\forall x \neg (Fx \wedge \neg Gx)$ , with ' $\supset$ ' for material implication.

Just as the indefinite part «someone» in Stoic existential propositions is non-referring (above), so are the indefinite parts «some one – that one». The relation between the existential and its correlated definite proposition has a parallel in the relation between the universal and its subordinated proper conditionals with the same referent for the subject expression in both component propositions. In the existential case, for truth, the presence of a true correlated definite is required. In the universal case, for truth, the absence of (the possibility of) a false subordinated conditional is required.

We now return to Quine's 'Variables Explained Away'. Quine notes that in simple monadic existential predicates (corresponding to one occurrence of the variable), the variable can be dropped, and that 'some (*G*) is *F*' is a better formulation than 'some (*G*) *x* is such that *x* is *F*' (343). On the Stoic side, for such cases, the Scope Principle does all that is necessary. Position is marked by the indefinite part 'someone/something', and the desired monadic predicate is abstracted as being everything other than this part, as per the definition of indefinite propositions. So the Stoic 'someone (or something) *F*' (for short  $\tau F$ , with ' $\tau$ ' for *tis*, *tinis*, etc.) combines the existence prefix with the variable it binds – in modern terms '(something *x* is such that *x*) *F*'. For Stoic monadic universal predicates, in variable-binding quantifier formulations, the same variable occurs twice. Here the Scope Principle combined with anaphoric reference does all that is necessary. '(If) someone – that one' combines marking the positions of (what would be) the two occurrences of the variable (one position is given by 'someone', the other by 'that one') with the analogue to the universality prefix 'everything *x* is such that'. We give the form of sentences expressing such propositions as  $\tau_i F \rightarrow \varepsilon_j G$ , where the predicate comes in the three parts  $\rightarrow$ , **F** and **G**, with ' $\varepsilon$ ' for the anaphoric expressions 'he' (*ekeinos*),

(below §§6 and 9). (ii) We believe the texts suggest that universal quantification has largest scope (below §7).

she (*ekeinê*), etc., and the anaphoric relation indicated by subscripts  $'_i'$ ,  $'_k'$ , .... Generally, in the two Stoic monadic cases, a combination of the Scope Principle and anaphoric reference suffices. Problems of multiple generality can occur only in indefinite propositions with more than one quantification.

### 6. Variable-free predicate logic: Polyadic predicate logic

To see whether Stoic logic could handle the problem of multiple generality, we need to consider propositions with more than one quantifying expression. In keeping with the evidence, we confine ourselves to dyadic predicates and propositions with two quantifying expressions. There are four such cases, in terms of modern quantifier expressions:  $\exists\exists$ ,  $\forall\forall$ ,  $\exists\forall$ ,  $\forall\exists$ . All issues of ambiguities are postponed to §7 and §8.

For the first two cases, consider again the Stoic if-clause donkey sentence from text (I). For our purpose it is preferable, here and below, to retain the Greek word order, even though in English it is unidiomatic and potentially cringe-inducing:

(26) «If someone (male or female) to something gave birth, then she of it is the mother».<sup>93</sup>

The 'antecedent' provides us with a Stoic  $\exists\exists$  proposition that contains a dyadic predicate (less-than-a-katêgorêma):

(27) «Someone to something gave birth.»

This appears to be the canonical or regimented positioning of the indefinite parts, parallel to that of the upright and oblique case-contents in middle propositions with dyadic predicates (§4). With  $F$  for 'gave birth to', the form of (27) is analogous to

(28)  $\exists x\exists y Fxy$   $\tau_1/\tau_2 F$

We have an explicit account of the Stoic truth-conditions for indefinites only for (affirmative) *monadic existential indefinite propositions*:

93. DL 7.75: εἴ τις τι ἔτεκεν, ἐκείνη ἐκείνου μήτηρ ἐστί.

'Someone is walking' is true precisely if the corresponding definite proposition is true (SE *M* 8.98; §5 above; Crivelli 1994a, 190–193). By simple generalization, we obtain convincing Stoic truth-conditions for dyadic (double) *existential* indefinite propositions, by which we mean those that contain a dyadic predicate and two indefinite expressions. The multiply general «Someone loves someone» is true if the corresponding doubly definite «This one (*indicating a specific person*) loves this one (*indicating a specific person*)» is true. Here there are two acts of indicating directed at two different people (cf. PHerc 307 Col. IV), or possibly at the same person. We use  $d, e, \dots$  as individual constants in demonstrative atomic propositions that are accompanied by the relevant admissible acts of indicating. Then, in contemporary terms,  $\exists x\exists y Fxy$  is true iff some  $Fde$  is true.

What about the Stoic dyadic universals or  $\forall\forall$  cases? It is realistic to assume that in (26) «... of --- is the mother» (using Greek word-order again) is also a dyadic predicate (less-than-a-katêgorêma).<sup>94</sup> As a whole, (26) then illustrates the Stoic version of a dyadic predicate with two restricted *universal* quantifications.<sup>95</sup> The predicate «If ... to ---gave birth, ... of --- is the mother» is in form analogous to

(29)  $Fxy \rightarrow Gxy \dots /---F \rightarrow \dots /---G$

The *proposition* (26) includes two pairs of quantifying parts, 'someone-she' and 'something-it', that take the two pairs of argument places. This is precisely what we expect from what we know about the monadic cases.<sup>96</sup> The form of (26) is then analogous to

94. Note that the main clause 'that one is mother to another one' contains two anaphoric expressions without their referent-expressions, and as such does *not* express a Stoic proposition. It is not a complete content.

95. There are no examples of unrestricted universalization with two-place predicates (analogues to  $\forall x\forall y Fxy$ ). However, recall the universal proposition at SE *M* 11.11, discussed above in §5 and below in §7: «if something exists, then it is either good or bad or indifferent».

96. It also gives additional support to the DL 7.70 reading of 'someone' and 'he' (*ekeinos*) above in §2.

$$(30) \forall x \forall y (Fxy \rightarrow Gxy)^{97}$$

It is not clear if and how such propositions could be expressed with ‘every’ (as ‘if something is human, it is mortal’ was said to be synonymous to ‘(every) human is mortal’ in the monadic case in §5) – not even if one resorts to acrobatic distortions of natural language.<sup>98</sup> Thus here we have a *logically motivated* reason why the Stoics introduced the conditional form for expressing universal propositions: it is *required* when there are two universalizations in a proposition. Of course (26) is not what one would expect the restricted parallel to

$$(31) \forall x \forall y Fxy$$

to be. Stoic dyadic predicates provide the material for a restricted version of (31), too. An example would be

(32) If someone <sub>$\tau_1$</sub>  is human and someone <sub>$\tau_2$</sub>  is human, then they <sub>$\epsilon_1$</sub>  love them <sub>$\epsilon_2$</sub> .<sup>99</sup>

$$(33) \forall x \forall y ((Fx \wedge Fy) \rightarrow Gxy)$$

Or, with the existence predicate  $E$ ,

(33a) If someone <sub>$\tau_1$</sub>  exists and someone <sub>$\tau_2$</sub>  exists, then they <sub>$\epsilon_1$</sub>  love them <sub>$\epsilon_2$</sub> .

$$(34) \forall x \forall y ((Ex \wedge Ey) \rightarrow Gxy)$$

97. In our Stoicized symbolism:  $\tau_i/\tau_k F \rightarrow \epsilon_i/\epsilon_k G$ , with the regimented word order and  $\tau_i$  for the first half of the indefinite (upright) part,  $\epsilon_i$  for the corresponding anaphoric second half,  $\tau_k$  for the first half of the indefinite oblique part and  $\epsilon_k$  for the corresponding anaphoric oblique second half. It is of course not necessary that an upright first half has an *upright* anaphoric part, or an oblique first half an *oblique* anaphoric part (see §8).

98. Cf. the well-known criticisms of the *suppositio* theories developed by Medieval logicians: Dummett 1973, 8, and on ancient Aristotelian logic, see Barnes 2007, 159–165.

99. Perhaps as a regimented form for ‘some are human’ (τινες ἄνθρωποι εἰσιν), given Chrysippus’ interest in plurals, see fn. to (48).

(32) and (33a) can be expressed with ‘every’ in Greek as in English, e.g. ‘every human loves every human’.<sup>100</sup>

We saw that there are no explicit accounts of the truth-conditions for Stoic *monadic universal* propositions, analogous in form to  $\forall x(Fx \rightarrow Gx)$ , but that there is sufficient evidence to reconstruct these: A monadic universal indefinite proposition is true precisely when there can be no false subordinate instances (§5). By generalization to dyadic universal indefinite propositions, a Stoic proposition analogous in form to  $\forall x \forall y(Fxy \rightarrow Gxy)$  should be true precisely when there can be no false subordinate instances of these: No  $Fab \rightarrow Gab$  can be false. In addition, we have an explicit semantic presentation of a counterexample invoked to show the falsehood of the plausible proposition (26).

(O) But this is false. For it is not the case that the hen of an egg is the mother.<sup>101</sup>

Here we have, implied, a sufficient condition for the falsehood of (26): Given the truth-conditions for indefinite conditionals (see Barnes et al. 1999, 112–113; SE M 11.9–11), (26) is false because one of its subordinated non-indefinite conditionals is false, presumably: «if a hen to an egg gives birth, then a hen to an egg is a mother», following the example in (O). A necessary and sufficient condition for this Chrysippean genuine conditional to be true is that it cannot have a true antecedent and a false consequent.<sup>102</sup> This suggests that the truth-conditions for (26) are that it cannot have a counterexample; and that (O) gives a counterexample in the form of a false subordinated in-

100. πᾶς ἄνθρωπος πάντα ἄνθρωπον φιλεῖ. Sentences expressing propositions like (32) and (33a) are known as *bishop sentences*, after Kamp (see Heim 1990 and, for a brief exposition, Elbourne 2010, 65–68). Since no Stoic examples of this kind survive, we mention this only in passing and do not speculate about the Stoic take on their semantics.

101. DL 7.75, continuation of (I): ψευδός δὲ τοῦτο· οὐ γὰρ ἡ ὄρνις ψοῦ ἐστι μήτηρ.

102. Barnes et al. 1999, 106–108. We saw that for the truth of an indefinite proposition we may need the truth of a corresponding *definite* proposition (or partially definite proposition). As expected, for the demonstration by counterexample of the falsehood of a proposition, a corresponding *middle* proposition seems to suffice.

stance. We assume that the same would be the case for propositions like (32) and (33a).

### 7. Scope ambiguity

The kind of ambiguity that is standardly offered as evidence of the ingenuity of Frege's quantifier logic is that of *scope ambiguity* in quantifying expressions (e.g. Dummett 1973, 9–15). Since the order of two or more of the *same* quantifying expressions is considered immaterial in every logical respect (though see §8), such ambiguity is discerned when quantifying expressions are mixed.<sup>103</sup> The most basic contemporary forms are  $\forall y \exists x Fyx$  and  $\exists x \forall y Fyx$ . The problem of multiple generality is often explained with examples of just these forms. 'Everyone loves someone' is the paradigm in the English language. Here 'everyone' may have wide or narrow scope. There are no surviving examples of Stoic propositions with mixed quantification, that is, of polyadic predicates that are governed by a mixture of existential and universal indefinite parts. At the same time, there is *no reason* to think that Stoic dyadic predicates exclude completions that generate propositions with mixed indefinite quantification. The only proviso is that, as in the case of monadic predicates, universal 'quantifying' is usually restricted, and Stoic language regimentation would ask for expression in conditional form. Using what information there is on Stoic monadic existential and restricted universal quantifying, it is straightforward to (re)construct step by step the Stoic version of the mixed cases. The Greek for 'Every human being loves someone' has a scope ambiguity similar to the English. In this section we disregard anaphoric ambiguity. Our examples are deliberately chosen so as to avoid that issue. Thus ancient Greek contains sentences equivalent in form to

(35) Every man likes something.

103. In Stoic notation of dyadic universal propositions, it is not immaterial which second half of a two-part quantifying expression goes with which first half. See §§6 and 8.

which start with the Greek for 'every' (*pas*). ((35) restricts only the universal, not the existential part.) Assume such a sentence is intended to express a Stoic proposition in which the universal part has wide scope. Then, in principle, the formulation with 'every' (*pas*) as first word would find Stoic approval because of their Scope Principle, that the first expression of a sentence indicates widest scope (see §1).<sup>104</sup> Possibly because of the perceived ambiguity in (35) between Chrysippean and 'Philonian' universal (§6), the Stoics would advocate regimentation. For brevity, we here consider only the Chrysippean versions. So, the Stoic fully regimented form of such sentences as *Chrysippean universals* would almost certainly be:

(36) If someone is a man, he likes something.<sup>105</sup>

If someone<sub>τ1</sub> is a man, he<sub>ε1</sub> likes something<sub>τ2</sub>.

This gives wide scope to 'someone-he' (*tis-ekinos*), before 'something' (*ti*). The similarity to the contemporary formal analogue is obvious:

(37)  $\forall y (Fy \rightarrow \exists x Gyx)$

104. OBJECTION: There are no attested examples of Stoic sentences with *pas* as the subject expression. Therefore these generally do not seem to be cases that the Stoics were interested in or had dealt with. REPLY: This objection is unconvincing. We assume that it was the Stoic view that universal propositions should be expressed in the form of one of two specific types of indefinite sentences, since the Stoics took universal sentences to be ambiguous, and that they probably regimented such sentences to disambiguate them. Depending on the intended meaning, they were to be formulated as indefinite conjunctions or indefinite conditionals. (As main evidence we take Cic. *Fat.* 11–15, also DL 7.82 (Sorites), Augustine, *Civ.* 5.1; for further evidence, see Bobzien 1998, 156–167, and the discussion in Frede 1974, 101–106). So it is not the case that the Stoics were not interested in sentences with *pas*. Rather, they found such sentences lacking, since ambiguous, and introduced a disambiguating regimentation, presumably for purposes of scientific theory and dialectic. It would hence be *surprising*, if we found such sentences as examples in Stoic logic, and their absence cannot be taken as evidence that the Stoics were not interested in the content of what they thought was intended to be expressed in natural Greek language in sentences with *pas*.

105. εἴ τις ἐστὶν ἄνθρωπος, ἐκεῖνος τι φιλεῖ. Cf. SE M 11.8–9.

Next assume that (the Greek equivalent to) (35) is intended to express a Stoic proposition in which the existential part has wide scope. The Scope Principle suggests that, in this case, the formulation of (35) would not even in principle find Stoic approval – or at least not for logical purposes. The immense flexibility of word order in Greek seems to stop short of allowing alternatives that start the sentence with an existential particle followed by a universal particle (*ti pas*, say). We are lucky to have *some* evidence that guides us in reconstructing how the Stoics would have regimented such sentences with wide-scope existential. It is based on a parallel case in which the Stoics introduce an existential proposition as part of their parsing a *simple* middle or definite proposition: ‘Kallias is walking’ is parsed as ‘There exists a certain Kallias who is walking’ and ‘Kallias is not walking’ as

(38) ‘There exists a certain Kallias who is not walking’.<sup>106</sup>

(The Stoic point of this rephrasing is to remove the scope ambiguity of negation by making it explicit that Stoic affirmative definite and middle propositions, which include «Kallias is not walking», have existential import. The Stoic negation «not: Kallias is walking» does not.) Here the structure of a *simple* middle proposition is made apparent by parsing the structure in a sentence that consists of a combination of

106. See Alex. An. Pr. 402.15–18, reporting a Stoic view: ‘for they say if Kallias does not exist, «Kallias is not walking» is no less false than «Kallias is walking», since in both what is signified is that a certain Kallias exists and that walking (or not-walking) holds of him’ (μη γὰρ ὄντος Καλλίου οὐδὲν ἥττον φασὶ τῆς «Καλλίας περιπατεῖ» ψευδῆ εἶναι τὴν «Καλλίας οὐ περιπατεῖ» · ἐν ἀμφοτέροις γὰρ αὐταῖς εἶναι τὸ σημαινόμενον ἔστι τις Καλλίας, τούτῳ δὲ ὑπάρχει ἢ τὸ περιπατεῖν ἢ τὸ μὴ περιπατεῖν). Alexander’s phrasing ‘that walking (or not-walking) holds of him’ is *Peripatetic*. We think that the formulation of two Stoic examples with *relative clauses* later in the passage is the canonically Stoic one (Alex. An. Pr. 402.29–33): ‘so thus the one saying “this one isn’t walking” says what is equivalent to “there exists this one indicated here, who isn’t walking” ... the one saying “the teacher Kallias isn’t walking” says what is equivalent to “there exists a certain teacher Kallias who isn’t walking”’ (οὕτως γὰρ τὸν λέγοντα ‘οὗτος οὐ περιπατεῖ’ ἴσον λέγειν τῷ ‘ἔστιν ὁ δεικνύμενος οὗτος, ὃς οὐ περιπατεῖ’ ... τὸν λέγοντα ‘Καλλίας ὁ γραμματικός οὐ περιπατεῖ’ ἴσον λέγειν τῷ ‘ἔστι τις Καλλίας γραμματικός, ὃς οὐ περιπατεῖ’). See also Alex. An. Pr. 404.27–29. On Alex. An. Pr. pp. 402–404, see also Lloyd 1978.

two clauses. We suggest that in our case of mixed quantified sentences with wide-scope existential, Stoic logicians may have used a parallel Greek formulation with an indefinite particle instead of a noun or demonstrative pronoun as a step towards regimenting the mixed sentences:

(39) There exists something that every man likes (*esti ti ho pas anthrōpos philei*).

This sort of formulation ‘there exists something that ...’ comes as close to an existentially quantifying expression in natural language as one may wish for. The Scope Principle gives it wide scope in (39). It is also very hard, if not impossible, to read such sentences as having ‘every’ with wide scope. The Stoics recommend (38) as parsing of the simple proposition ‘Kallias is not walking’ in the context of *evading scope ambiguity*, i.e. of negation.<sup>107</sup> It is thus entirely plausible that they also recommend formulations like (39) as parsing of a simple indefinite proposition to evade scope ambiguity (cf. Egli 2000).

Some of the Stoic discussion of existential import in simple affirmative propositions suggests a parsing of (38) as a ‘conjunction’ with an anaphoric expression like this one

(40) Socrates exists and he (that one) is walking.<sup>108</sup>

This shows that the Stoics had all the elements available to parse sentences like (39) as indefinite conjunctions<sup>109</sup> that have an existential first conjunct and an indefinite conditional, or Chrysippean universal, as second conjunct:

(41) There exists something (*esti ti*), and if someone is a man, he likes it.

107. See previous note.

108. ἔτι ὁ λέγων ‘Σωκράτης περιπατεῖ’ ἴσον λέγει τῷ ‘ἔστι τις Σωκράτης, κάκεινος περιπατεῖ’ (Alex. An. Pr. 404. 27–28).

109. See Cic. Fat. 15 and discussion in Bobzien 1998, 156–159; Crivelli 1994a, 201–202.

The parallel with (40) shows that the formulation with an initial existential ‘conjunct’ is not conjured out of thin air. Parallel to the case of (40), the parsing of (35) with wide-scope existential as (41) gives us the structure of the mixed existential-wide-scope proposition with restricted universal *without making it a genuine Stoic conjunction or a conditional*. Again, the similarity to contemporary analogues with restriction is obvious. Depending on whether existence was considered a predicate (with *Ex* for ‘*x* exists’),<sup>110</sup> they correspond to one of these:

$$(42) \exists x \forall y (Fy \rightarrow Gyx)$$

$$(43) \exists x (Ex \wedge \forall y (Fy \rightarrow Gyx))$$

So, the Stoics are able to indicate the fact that the existential part has wide scope in the proposition both with the non-‘conditional’ sentence plus Scope Principle (39) and with the ‘conditional’ expression of the universal (41).

Based on the semantics for *monadic* indefinite propositions, we can offer a reconstruction of the semantics of Stoic *dyadic* indefinite mixed propositions. For the semantics of those with wide-scope existential, we combine Stoic truth-conditions for the existential with those for the universal, in order. The proposition (41) is true, if a corresponding definite proposition is true. The corresponding definite conjunction has two occurrences of a definite pronoun that includes indicating a particular thing twice (‘this<sub>δ1</sub>’), in lieu of ‘something’ and ‘that thing’:

$$(44) \text{Both this}_{\delta 1} \text{ exists and if someone is a man, he likes this}_{\delta 1}.$$

110. We do not here discuss the question whether in these cases the Stoics consider ‘exists’ (εἶναι / ὄντα) as a predicate. It appears, though, that they do. Chrysippus thinks the claim that ‘of all things that exist, some are good, some bad, some in between’ is equivalent to the universal (indefinite conditional) claim, ‘if some things exist, they are either good or bad or indifferent’ (τῶν ὄντων τὰ μὲν ἔστιν ἀγαθὰ, τὰ δὲ κακά, τὰ δὲ τούτων μεταξύ δυνάμει κατὰ τὸν Χρυσίππρον τοιοῦτόν ἐστι καθολικόν· εἴ τινὰ ἔστιν ὄντα, ἐκεῖνα ἦτοι ἀγαθὰ ἔστιν ἢ κακά ἔστιν ἢ ἀδιάφορα) (SE M 11.11, mentioned in §5). Note also another regular example in Stoic logic: ‘It is day’, which translates literally as ‘day exists’ (ἡμέρα ἔστιν, DL 7.68 *et passim*).

(44) is a *proper* Stoic conjunction with a Stoic indefinite conditional as second conjunct. (It can be ‘cut’ – see §9.) For (44) to be true, both its conjuncts need to be true. The semantics for the first conjunct are clear (§5, fn. 75). So are those for the second, which is a monadic universal indefinite proposition (above §6). The indefinite conditional is true when no corresponding definite conditional can be false. So (41) as a whole is true, when the proposition obtained by replacing the indefinite conditional in (44) with a definite one that has two occurrences of a definite pronoun that includes indicating a particular thing twice (‘this<sub>δ2</sub>’) cannot be false:

$$(45) \text{Both this}_{\delta 1} \text{ thing exists and if this}_{\delta 2} \text{ one is a man, this}_{\delta 2} \text{ one likes this}_{\delta 1} \text{ thing.}$$

So the Stoic semantics of the propositions of kind (41) is straightforward.

For the semantics of propositions with wide scope – Chrysippean – universal (36) (‘If someone is a man, he likes something’) truth-conditions may not fall into place as easily. Even so, it is clear that they would logically differ from the narrow-scope one. The proposition expressed by (36) is true if it *cannot have* false subordinate cases. Subordinate cases are of the kind

$$(46) \text{If this one}_{\delta 1} \text{ is a man, this one}_{\delta 1} \text{ likes something.}$$

These are proper conditionals and false if it *can be* that their antecedent is true and their consequent false. So every subordinate case like (46) *must be* true. Proposition (46) is true if it *cannot be* the case that «this one<sub>δ1</sub> is a man» is true and «this one<sub>δ1</sub> likes something» is false. So we *cannot* have that. In other words, we *cannot* have «this one<sub>δ1</sub> is a man» true and «not: this one<sub>δ1</sub> likes something» true. Hence we *cannot* have any proposition be true that is of the kind

$$(47) \text{Both this one}_{\delta 1} \text{ is a man and not: this one}_{\delta 1} \text{ likes something.}$$

(For the corresponding semantics of propositions with wide-scope ‘Philonian’ universal, simply replace ‘cannot have’ with ‘has no’, ‘can be’ with ‘is’, ‘must be’ with ‘is’, etc., and mutatis mutandis below.)

As for the logical relations between the two kinds of dyadic mixed indefinites, we expect proposition (36) to be true whenever proposition (41) is true, but not vice versa. This is indeed the case. If there can be no counterexample to (36), there can be none to (41). Moreover, (36) can be true when (41) is not. In (36) ‘something’ is not anaphoric, whereas in (41) ‘it’ is anaphoric on ‘something’ (*ti*). And in the definite conditional (45) that corresponds to (41), ‘this thing’ has twice the same demonstrated object, whereas for (36) there is no such double-demonstration requirement.

The relevant logical relation between the cases  $\forall\forall$  and  $\forall\exists$  (see introduction) can also be expressed. Take for  $\forall\forall$  proposition (32) from §6. (32) is true when there can be no counterexample (of two humans who don’t love each other, i.e.):

(48) Both (both this one <sub>$\delta_1$</sub>  is human and this one <sub>$\delta_2$</sub>  is human)<sup>111</sup> and not: (this one <sub>$\delta_1$</sub>  loves this one <sub>$\delta_2$</sub> ).

Take for  $\forall\exists$

(49) If someone <sub>$\tau_1$</sub>  is human, then that one <sub>$\epsilon_1$</sub>  loves someone <sub>$\tau_2$</sub>  and that one <sub>$\epsilon_2$</sub>  is human.

Proposition (49) is true when there cannot be a counterexample (of a human who doesn’t love a human, i.e.):

(50) Both this one <sub>$\delta_1$</sub>  is human and not: (both this one <sub>$\delta_1$</sub>  loves someone <sub>$\tau_2$</sub>  and that one <sub>$\epsilon_2$</sub>  is human).

Clearly, if there can be no counterexample (48) to (32), then there can be no counterexample (50) to (49).

111. This would corroborate, with an additional case, the Stoic regimentation of the plural ( $\sigma\upsilon\tau\omicron\iota\ \acute{\alpha}\nu\theta\rho\omega\pi\omicron\iota\ \epsilon\iota\sigma\omega$ ). Cf. the example in SE *M* 10.99 and the extended discussion in Crivelli 1994b. See also PHerc 307 Col. IV for a case of the speaker expressing a definite proposition by means of pointing at two people.

In sum, the Stoics thus had all the means available to distinguish both syntactically and semantically between the two basic kinds of mixed multiple generality, and without variables. Within Stoic logic of predicates and indefinite propositions, the paradigm scope ambiguity of mixed-quantifier propositions with dyadic predicates (and restriction on universal propositions) can be removed. This is achieved (i) by language regimentation with the Scope Principle for sentences with ‘every’ (*pas*) and ‘someone’ (*tis*) and (ii) at the level of parsing propositions, by the parsing of universal propositions into the form of indefinite conditionals or indefinite negated conjunctions. Thus the Stoics are able to express unambiguously (and without variables) multiple generality for a language with restricted universality — in the absence of any anaphoric ambiguity.

### 8. Anaphoric ambiguity in Stoic quantifying expressions

Fregean variable-binding quantifiers eliminate two kinds of ambiguity: scope ambiguity and anaphoric ambiguity.<sup>112</sup> In a proposition (or more generally complete content), quantification with variables unambiguously binds every occurrence of every variable, and anaphoric ambiguity does not occur. The Stoics, though, have no variables. Does or can Stoic logic account for anaphoric ambiguity? Consider the following generic stripped-down (contemporary) form of the Stoic donkey sentence that expresses proposition (26), with two universal quantifying expression that include two occurrences of anaphora

(51) If someone someone *F*, then that one that one *G*.

Such sentences are potentially ambiguous between

(52) If someone<sub>1</sub> someone<sub>2</sub> *F*, then that one<sub>1</sub> that one<sub>2</sub> *G*.

(53) If someone<sub>1</sub> someone<sub>2</sub> *F*, then that one<sub>2</sub> that one<sub>1</sub> *G*.

112. We here use ‘anaphoric ambiguity’ as short for ‘context sensitivity of anaphoric expressions that cross-refer to non-referring expressions within the same sentence, considered in isolation, with the context being a linguistic one’.

Again,  $'_1'$  and  $'_2'$  are used to indicate which anaphoric expression refers to which indefinite part(icle). With two indefinite 'someone' particles followed by two anaphoric 'that one' particles, it is ambiguous which anaphora is bound to which particle. Similar ambiguities can be observed in mixed quantification.

In their logic, the Stoics have at their disposal four elements that can in principle play the roles of operators of the kind Quine introduces: (i) position or order of Stoic quantifying expressions, (ii) active-passive formulations, (iii) declension and (iv) gender. It has baffled historians of logic that the Stoics consider all four at the level of Stoic propositions, as contrasted with linguistic items.<sup>113</sup> Stoic theory of (variable-free) quantification may help explain why. We consider different cases.

For two existential quantifiers, our existing examples are 'someone gives birth to someone' and, implied, 'someone is someone's mother'. In principle, there can be ambiguity. If we add indices to the indefinite expressions, we have

(54) Someone<sub>1</sub> someone<sub>2</sub> gives birth to.

What determines, informally speaking, who gives birth to whom? Word position is extremely flexible in Greek. Nonetheless we have seen that the Stoics regiment position. Another disambiguating element is declension. In (54), declension is sufficient in the Greek, since the nominative and dative pronouns would differ (e.g. 'someone', 'to someone'). The Scope Principle may have required that the upright case (*orthê ptôsis*) be placed first, thereby determining what kind of proposition we have. This is consistent with all our examples for indefinite propositions. Declension does, however, not work in cases of equality and identity, for example

(55) Someone<sub>1</sub> someone<sub>2</sub> is.

113. See e.g. Frede 1994. Some have simply assumed they must be functions of language (ibid. 13–14).

Stoic corresponding middle and definite propositions are «this one  $\delta_1$  is this one  $\delta_2$ » and «Dio is Theon».<sup>114</sup> In (55) declension is of no use, and position the only option for disambiguation. Alternatively one might argue that, due to symmetry *combined with indefiniteness*, there is no ambiguity, since exchanging 'someone<sub>1</sub>' and 'someone<sub>2</sub>' does not affect what is said. (However, we here reserve judgement on the question whether «this one  $\delta_1$  is this one  $\delta_2$ » is the same proposition as «this one  $\delta_2$  is this one  $\delta_1$ » and «Dio is Theon» the same as «Theon is Dio», as opposed to simply being equivalent.)

The proposition (26) (from §§6 and 7) is an example with *two pairs of universally quantifying parts*. (52) and (53) show that and how such sentences are potentially ambiguous. In sentence (26), gender indicators avert ambiguity.<sup>115</sup> We use subscript indices,  $_{M'}$   $_{F'}$   $_{MF}$  and  $_N$  as indicators of male, female, male or female, and neuter gender to express this for (26):

(56) If someone<sub>MF</sub> to something<sub>N</sub> gives birth, then that one<sub>F</sub> to that thing<sub>N</sub> is mother.

Gender then works similarly to *selectional restriction*. But gender does not always prevent ambiguity. Take the following case:

(57) If someone<sub>MF</sub> to someone<sub>MF</sub> gives birth, then that one<sub>F</sub> to that one<sub>F</sub> is mother.<sup>116</sup>

114. ἔστιν οὗτος οὗτος, εἶναι τοῦτον τοῦτον and Δίων Θεών ἐστιν, Δίωνα Θεώνα εἶναι, cf. PHerc 307 Cols IV and V. It is also found in Galen, *On Fallacies* 4.7–9, for which see discussion in Edlow 2017, 64–65, and Atherton 1993, 383.

115. Some Stoics considered the gender of a demonstrative case-content to be semantically relevant and thus part of a proposition: «this one  $\delta_M$  is walking» and «this one  $\delta_M$  is not walking» are both false (false by *paremphasis*, *Alex. An. Pr.* 402.25–26), if the object of the act of indicating (*deixis*) is female. These two propositions are said to be equivalent to «this one  $\delta_{M'}$  who is being indicated, exists and is (not) walking». For discussion, see Durand 2019, §§21–24. Cf. *Alex. An. Pr.* 402, 21–30, cited in §7 above. See also *Alex. An. Pr.* 404.31–34.

116. Here and in the following sentences (58), (59), (60), (61), (58a) and (59a), the feminine gender in the 'consequent' (*ekeinê*, ...) disambiguates the generic, feminine or masculine, gender in the 'antecedent' (*tis*, ...) as feminine. Our

This is ambiguous in that it could denote either of the following un-regimented propositions

(58) If someone<sub>MF1</sub> to someone<sub>MF2</sub> gives birth, then that one<sub>F1</sub> to that one<sub>F2</sub> is mother.

(59) If someone<sub>MF1</sub> to someone<sub>MF2</sub> gives birth, then that one<sub>F2</sub> to that one<sub>F1</sub> is mother.

Here gender and declension do not help; neither does position alone. We know that *de facto* listeners will, by considerations of charity, choose the interpretation that makes the sentence true (e.g. Davidson 1974, Stalnaker 1978, Grice 1989). However, from a Stoic perspective, such pragmatic considerations would not be part of logic – nor is such information always semantically or contextually provided. This leaves active and passive. Chrysippus wrote a book on active and passive predicates, and discussed them elsewhere.<sup>117</sup> Perhaps the following passage gives us his view. In any case, it gives a Stoic view.

(P) Of (monadic) predicates, some are active (*ortha*), some passive (*huptia*), and some neither. Active are those that, being connected with one of the oblique case-contents (*ptôseis*), yield a monadic predicate, for example ‘hears’, ‘sees’, ‘is conversing with’.<sup>118</sup> Passive are those that, being connected with a passive particle [yield a monadic predicate], for example ‘am heard’, ‘am seen’.<sup>119</sup> (DL 7.64)

subscript numerals determine the anaphora relation between the pronouns independent of their specific gender indicators.

117. DL 7.192, Chrysippus, *Log. Inv. fr.* 3 lines 4–18; Col. I.23, II.17–21, all noted by Barnes 1999, 206 and Marrone 1984.

118. We read this as saying that an active predicate comes to be by the connection with an oblique case-content, in the sense that the active predicate is the monadic predicate that is the result of this combination. So also Barnes 1999; Hülser 1987–88, 809; Hicks 1925; Mensch 2018.

119. DL 7.64: καὶ τὰ μὲν ἐστὶ τῶν κατηγορημάτων ὀρθά, ἃ δ’ ὕπτια, ἃ δ’ οὐδέτερα. ὀρθὰ μὲν οὖν ἐστὶ τὰ συντασσόμενα μὲν τῶν πλαγίων πτώσεων πρὸς κατηγορήματος

Stoic active predicates have by definition as content element an oblique case-content, i.e. that which would be expressed by a declined noun expression. The examples for the passive employ two of the three verbs that are given for the active. This, together with the talk of a passive particle,<sup>120</sup> suggests that the Stoics may have thought that one can convert one to the other. (A later text says this explicitly.<sup>121</sup>) We may think of the passive particle as something like a *transformation operator* that, applied to the active, provides the passive (at the content level).<sup>122</sup> A combination of *Active-to-passive Conversion* with the Scope Principle positioning allows the removal of the ambiguity in (57):<sup>123</sup>

(60) If someone<sub>MF1</sub> to someone<sub>MF2</sub> gives birth, then that one<sub>F1</sub> to that one<sub>F2</sub> is mother.

(61) If someone<sub>MF1</sub> by someone<sub>MF2</sub> is born, then that one<sub>F1</sub> to that one<sub>F2</sub> is mother.

A *Principle of Anaphoric Congruence* that anaphoric reference is in order of occurrence (the first indefinite expression has the first anaphoric expression referring to it; the second indefinite expression has the second anaphoric expression referring to it) can then supplement the Scope Principle as a second principle for position. Thus position together with Active-to-passive conversion determines the ‘referent’ of the anaphoric expression. (57) is disambiguated into (60) and (61) by use of Anaphoric Congruence with Active-to-passive Conversion. Strictly speaking, in this example the Principle of Anaphoric

γένεσιν, οἷον Ἀκούει, Ὀρθά, Διαλέγεται· ὕπτια δ’ ἐστὶ τὰ συντασσόμενα τῷ παθητικῷ μορίῳ, οἷον Ἀκούομαι, Ὀρώμαι.

120. Presumably ὑπό, cf. Gaskin 1997, 92.

121. Scholium in Dionysius Thrax, 401.1–20. The ‘philosophers’ mentioned must be Stoic philosophers, since the terminology is Stoic.

122. Cf. Alex. *An. Pr.* 403.14–24, reporting Stoic views on the transformation (*enkklisis*) of present to past tense. So the Stoics were familiar with a notion of transformation of this kind.

123. Note the similarity of this approach to Peter Ludlow’s treatment of bishop sentences in Ludlow 1994, 171–172.

Congruence suffices without Active-to-passive Conversion, *if* one allows for variable case order, so that a nominative can take second place.<sup>124</sup> However, position alone does not suffice in examples with a neuter nominative and an accusative expression in the consequent clause, since these are generally identical in form. In contrast, *Active-to-passive Conversion* always allows reverse order of the antecedent expressions and thus can handle such examples:

(60a) If something<sub>N1</sub> something<sub>N2</sub> attacks, that thing<sub>N1</sub> (*ekeino*) that thing<sub>N2</sub> (*ekeino*) kills.

(60b) If something<sub>N1</sub> by something<sub>N2</sub> is attacked, that thing<sub>N2</sub> (*ekeino*) that thing<sub>N1</sub> (*ekeino*) kills.

Of course we do not know whether the Stoics suggested such disambiguation by use of Anaphoric Congruence with Active-to-passive Conversion. However, it does generally work for active and passive predicates. Assume that «... hears Dio» is an active monadic predicate (the result of connecting «... hears ---» with «Dio» as an oblique case-content (DL 7.64)), and that with «Plato» it forms the proposition «Plato hears Dio». Then an active-to-passive transformation yields the proposition «Dio is heard by Plato», formed from «Dio» and the passive monadic predicate «--- is heard by Plato». With Stoically regimented Greek word order, we get from 'Plato Dio hears' to 'Dio by Plato is heard' and thus, if required, can obtain the desired order of content-cases.<sup>125</sup> This may help explain why — unlike for Frege! (1897,

124. (58a) If someone<sub>MF1</sub> to someone<sub>MF2</sub> **F**, then that one<sub>F1</sub> to that one<sub>F2</sub> **G**.  
 (59a) If someone<sub>MF1</sub> to someone<sub>MF2</sub> **F**, then *to that* one<sub>F1</sub> that one<sub>F2</sub> **G**.

This provides an alternative for those who prefer a different reading of text (P), if at a cost. For the Stoics may have wished to keep the placement of cases for detachment in derivations (for examples, see the derivations discussed in §9). Instantiation becomes semi-automatic with *Active-to-passive Conversion*. Without, detachment either loses this feature or produces non-canonical formulations, e.g. 'to Hebe Hera is mother' rather than 'Hera to Hebe is mother'.

125. If the active content is a dyadic predicate (less than a katêgorêma), this works much the same. «... hears ---» connected with the oblique *ptôsis* «Dio» yields the monadic predicate «... hears Dio», which with the upright «Plato»

282) — active and passive are part of the content. It is not so much that e.g. 'Dio loves Plato' and 'Plato is loved by Dio', taken on their own, give us detectably different content. What is asserted with one does not differ from what is asserted with the other. Rather, it is that, *in certain more complex propositions*, the active and passive are structuring elements that are part of what structures the proposition, and thus are a detectable part of the simple proposition *in so far as* it is part of the more complex proposition. With (60a) 'someone<sub>F1</sub> to someone<sub>F2</sub> gives birth', the same proposition may be expressed (and asserted) as with (61a) 'someone<sub>F1</sub> by someone<sub>F2</sub> is born'. However, with the sentences (60) and (61), which contain (60a) and (61a) respectively, different complex propositions are expressed (and asserted). *If* the Stoics made use of their distinction of active and passive predicates in this way, we can observe a similarity to Dummett's distinction between *assertoric content* and *ingredient sense* (as e.g. set out in Dummett 1991, 47–50).<sup>126</sup>

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yields the proposition «Plato hears Dio». Then an active-to-passive transformation yields the proposition «Dio is heard by Plato», formed from «Dio» and the passive monadic predicate «--- is heard by Plato», which yields the dyadic predicate (less-than-katêgorêma) «--- is heard by ...» and the oblique case-content (*ptôsis*) «Plato».

126. How would this work for polyadic cases with an adicity higher than two? First note that natural languages like English generally have a hard time dealing with such cases when use is restricted to simple anaphoric expressions. Take the paradigm 'x is between y and z' as example: 'If something is located between something and something, it is neither the same as it nor the same as it.' The Stoic use, in their logic of ordinals for schematic representation of propositions in arguments and in their formulation of inference rules (*themata*), suggests that they would have resorted to numerals for such cases, as indeed English speakers might, too. The Stoics generally count each of the relevant elements and count them in order of occurrence (cf. DL 7.80–81 and Bobzien 2019). For the case at issue, this would yield 'if something is between something and something, then it is neither the second thing nor the third thing', and three-dimensionally, 'if something is between something and something and something, then it is neither the second thing nor the third thing nor the fourth thing', etc. Jointly, the Scope Principle and the Principle of Anaphoric Congruence are then sufficient for such cases, although there may be instances where some language regulation is required that goes against idiomatic formulations. Recall, though, that Greek is very flexible with word order.

Jointly, position, declension, gender and active-passive appear to suffice to resolve all anaphoric ambiguities that may occur within the variable-free framework of Stoic parsing of universal and existential quantification. For three-or-more-place predicates, the rules or operators based on active-passive, position, gender and declension at the level of content would have to be generalized for such predicates. We believe that this is to some extent possible.<sup>127</sup>

### 9. Polyadic predicates in Stoic theory of inference

Jonathan Barnes contends that the Stoic distinction between monadic and polyadic predicates ‘remained inert’ and that ‘no Stoic exploited the distinction in his account of inference and syllogism’ (Barnes 1999, 206). Stoic *syllogistic* or sequent logic (or proof theory or theory of deduction) does indeed not include the distinction between monadic and polyadic predicates. It is a propositional logic which does not analyse the content of atomic (i.e. simple affirmative) propositions (Bobzien 2019).

However, Stoic logic (and that includes inferences) was by no means restricted to Stoic syllogistic. We have evidence about the rudiments of a logic of imperatival inferences, of a modal logic, of tense logic, of a theory of suppositional inferences, of discussions of various logical paradoxes, of certain arguments (probably discussed in the context of the Liar) that included intensional expressions like ‘says’, and more.<sup>128</sup> And as should by now be clear, we have the rudiments of a variable-free predicate logic for monadic and dyadic predicates.

127. With this suggested reconstruction and elaboration on Stoic indefinite propositions, we can also answer the question how to determine whether, in the case of simple propositions with dyadic predicates and *mixed arguments* (definite, middle, indefinite), the propositions themselves are definite or middle or indefinite (Brunschwig 1994, 67). First a sentence that expresses such a proposition is put in canonical form. This requires the expression that signifies the subject argument to take first place, which, in the case of secondary event predicates (*parasumbamata*), would be linguistically expressed by the dative. The expression with the largest scope, according to the Scope Principle, then determines what kind of proposition we have.

128. For example: *Imperatives*: Barnes 1986, 21–26. *Modal logic*: Barnes et al.

There is also ample evidence that the Stoics acknowledge the validity of arguments (and argument forms or modes (*tropoi*)) that have a Stoic universal indefinite proposition as first premise, that have an instantiation of the ‘antecedent’ as second premise and that deduce as conclusion an instantiation of the ‘consequent’, where the same case-content, demonstrative or not, is instantiated in the second premise and the conclusion:

(62) If someone is walking, he is moving; this man is walking; therefore this man is moving. (So in Aug. *Dial.* 3,84–86; cf. Cic. *Fat.* 11–15)<sup>129</sup>

1999, 116–121 with 86–88; Gaskin 1995; Mignucci 1978; Frede 1974, 107–117; Kneale & Kneale 1962, 123–128; Mates 1961, 36–41. *Tense logic*: Denyer 1999; Gaskin 1995; Crivelli 1994b; Long & Sedley 1987, v.1, 51; Vuillemin 1985. *Suppositional inferences*: Bobzien 1997. *Nobody Paradox*: Mansfeld 1984; Caston 1999, 187–192. *Paradoxes of Presupposition*: Bobzien 2012. *Liar Paradox*: Barnes et al. 1999, 163–170; Cavini 1993. *Unmethodically conclusive arguments*: Barnes et al. 1999, 151–155.

129. QUESTION: Cannot (62) be taken to show that its first premise is in fact a Stoic conditional, that is, a non-simple proposition, since it seems that it could feature in a syllogism of the first indemonstrable form? REPLY: No. The Stoic first indemonstrable is described as being composed of a conditional and its antecedent as premises and its consequent as conclusion (DL 7.80, SE M 8.224). (62) is not of that form. FOLLOW-UP QUESTION: Can we not assume that the first premise in (62) is in fact a Stoic conditional, that is, a non-simple proposition, since in a first step the Stoics *infer from it* a definite or middle conditional and then generate a syllogism of the first indemonstrable form with *that* conditional? REPLY: There is no evidence that the Stoics had a valid inference form that permits an inference from an indefinite conditional to a corresponding definite or middle *conditional*. On the contrary, the evidence suggests that (62) as it stands was considered to illustrate a valid inference form. Both the Augustine passage and Cicero *Fat* 11–15 suggest that the Stoic inference went directly from the indefinite conditional and a definite or middle correspondent to its antecedent to the matching correspondent to its consequent. In addition we note that Stoic valid inference forms (Antipater’s controversial one-premise arguments aside) (i) all contain at least two premises; moreover (ii) it seems that the so-called wholly hypothetical syllogisms, which infer a conditional from two conditionals, were not accepted as syllogisms by the Stoics, cf. e.g. Bobzien 2000, so there is no precedent for the inference of a conditional from one or more conditionals of the same size (i.e. with the same number of two-place connectives). We believe then that we are on safe ground in assuming that (62) does not in any way show that, for

Moreover, there is evidence that the Stoics were specifically concerned with certain questions about inferences that involve indefinite propositions. First in line is the so-called Nobody paradox (*outis*), about which Chrysippus wrote eight books (DL 7.198). It is described as ‘an argument composed of an indefinite and a definite [proposition] that has its second premise and conclusion connected’ (DL 7.82). As most ancient paradoxes, it came in several variants. One is apparently ‘if someone is here, he is not in Rhodes; but a man is here; hence not: a man is in Rhodes’ (DL 7.82).<sup>130</sup> It is not hard to guess that the non-referring character of ‘someone’ and ‘nobody’, and the resulting restrictions on inference, played a central role in the discussion of this paradox. Second, in the same section of Chrysippean book titles on inferences, two books ‘on arguments from indefinite and definite [propositions]’ are sandwiched between titles of books on the Nobody (DL 7.198). This suggests their form is related to the form of the Nobody arguments. We assume that either (62) is an example of such ‘arguments [composed] from an indefinite and a definite [proposition] (i.e. as premises)’, or that they are of the more basic kind

(62a) ‘If someone is walking, he is moving; this one is walking; therefore this one is moving.’

(It is not unusual for Stoic illustrative arguments to be transmitted in slight variations.) Third, Chrysippus also wrote books entitled ‘Proofs that one must not cut the indefinites’ and ‘Reply to those that disagree

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the Stoics, indefinite conditionals (and indefinite negated conjunctions) were considered non-simple propositions.

130. Reading συντακτικός with Long: οὗτις δέ ἐστι λόγος συντακτικός ἐξ ἀορίστου καὶ ὠρισμένου, συνεστῶς πρόσληψιν δὲ καὶ ἐπιφορὰν ἔχων, οἷον “εἴ τις ἐστὶν ἐνταῦθα, οὐκ ἔστιν ἐκεῖνος ἐν Ρόδῳ. <ἀλλὰ μὴν ἐστὶ ἄνθρωπος ἐνταῦθα. οὐκ ἄρα ἄνθρωπός ἐστιν ἐν Ρόδῳ>” (DL 7.82). We choose this reading based on the parallel with DL 7.198 (see main text), and choose ‘a man’ (ἄνθρωπος) rather than Dorandi’s ‘someone’ (τίς) in the supplemented second premise and conclusion, since all other extant versions of the Nobody have ἄνθρωπος. For the variants of the Nobody, see e.g. DL 7.82 and 7.187; further evidence in Hülser 1987–88 as texts 1205–1207, 1209, 1247–1251; cf. also Mansfeld 1984; Caston 1999, 187–192.

with those who are against the cut of the indefinites’ (DL 7.197). Here ‘indefinites’ is likely to refer to indefinite conditionals and indefinite (negations of) conjunctions, and ‘to cut’ is likely to be a technical term. There are two plausible and related meanings: (i) One cannot cut an indefinite conditional into two component propositions. (ii) One cannot detach a conclusion. That is, from ‘if someone *F*, that one *G*’ one cannot detach, by using as second premise ‘someone *F*’, ‘that one *G*’ as conclusion. Since the titles are in the section of titles on inferences, the latter is more likely. There is no reason to assume that whatever is discussed in all these Chrysippean books is limited to indefinite propositions with *monadic* predicates.

Let us add to this indirect support for Stoic discussion of arguments with non-idle polyadic predicates that it is *implied* by (26) that the Stoics have inference schemata or argument forms that contain non-idle multiply generalized sentences. Here are text (I) (which contains (26)) and text (O) in succession:

(I) A proposition is plausible if it provokes assent, for example «If someone gave birth to something, then she is its mother». (O) But this is false. For the hen is not the mother of an egg. (DL 7.75)

This points toward the Stoics accepting as valid arguments such as:

(63) If someone to something gave birth, that one of that thing is the mother.

Now Hera to Hebe gave birth.

Hence Hera of Hebe is the mother.<sup>131</sup>

and:

(64) If someone to something gave birth, that one of that thing is the mother.

Now this hen to this egg gave birth.

Hence this hen of this egg is the mother.

131. The questions in fn. 129 could be rehashed here. The answers can as well.

More generally, it points toward their accepting the validity of argument forms like:

(65) If something<sub>1</sub> something<sub>2</sub> F, that thing<sub>1</sub> that thing<sub>2</sub> G.

Now **abF**.

Hence **abG**.

Why is this implied? It is implied since the hen-egg case is adduced as a counterexample to the truth of the indefinite conditional (26). But the hen-egg case produces such a counterexample *only because* there is an *assumed* valid argument form of the kind (64) that admits instantiation, with middle or definite propositions for ‘antecedent’ and ‘consequent’. We conclude that it is exceedingly likely that multiple generality was not inferentially idle in Stoic logic.

### 10. Conclusion

We have seen that the Stoics used regimented variable-free formulations for expressing existential and universal quantifying propositions with monadic and dyadic predicates. The structure of existential propositions with simple monadic predicates is determined by the positioning of the Stoic quantifying expression, which can be understood retrospectively as combining the existence prefix with the variable it binds. All universal propositions are parsed by the Stoics as having indefinite conditional form in which the indefinite part in the ‘consequent’ anaphorically cross-refers to the non-referring indefinite part in the ‘antecedent’. That is, they have the form of if-clause donkey sentences. (For ‘unrestricted’ universals, the ‘antecedent’ could have used an existence predicate.) The combination of rigid position with the parsing of universals as indefinite conditionals is adequate for the monadic cases.

The combination of rigid position and conditional formulation of universals without binding variables also suffices for cases of dyadic predicates, as long as there is no ambiguity introduced by the anaphoric expressions. Whether the Stoics introduced conditional

formulations in order to avoid the problems of multiple generality, or whether they were simply chosen to accurately represent the structure of propositions with monadic and dyadic predicates (including the different structures of their two kinds of universals), and the problems of multiple generality consequently just did not arise, we do not know. For cases of anaphoric ambiguity in propositions with polyadic predicates, rigid position and conditional formulation of universals are supplemented with case marking, gender marking and active-passive transformation, all of which the Stoics place at the level of content, and thus logic, as opposed to linguistic expressions and grammar.

At least for dyadic generality, the combination of case marking, gender marking and active-passive transformation with rigid position and conditional formulation of universals makes it possible to develop a system that covers the ground Fregean variable-binding quantifiers so niftily control. The Stoic placement of cases, gender and active-passive at the level of *content* is thus justified. The five factors converge into a method that ensures a system of equal strength as variable-binding quantifiers for dyadic predicates that can be expressed in (ancient Greek) natural language without undue deformations. The fact that Greek is case-based and has very flexible word order makes the addition of rules for rigid positioning easy. Some evidence implies that the Stoics considered argument forms that contain propositions with multiple generality, and that polyadic predicates were not inferentially idle in Stoic logic.

We have not offered a worked-out formal Stoic predicate logic. This would require an integration of Stoic indefinite propositions with their propositional logic. Neither have we ventured beyond dyadic predicates, although it seems to us that, up to a point, generalization to polyadic predicates is possible. Nor have we discussed the relation between Stoic logic and medieval and early Renaissance treatments of multiple generality.<sup>132</sup> Patently, there is space for further research.

<sup>132</sup>. We are no experts in medieval or early Renaissance philosophy. It seems to us, though, that the attempts at solving the problems of multiple generality by medieval philosophers are generally impeded by their commitment to

Our goal was to make a case for the claim that a logical treatment of multiple generality is found long before Frege, already in antiquity.

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Aristotelian term logic and to their notion of *suppositio*, so that solutions, even if possible, would be described *by them* in unnecessarily convoluted ways. We do not doubt that the syntax and semantics of medieval Latin could handle multiple generality, nor that Terence Parsons' ingenious *Linguish* (Parsons 2014) may reflect this fact, nor that some cases of multiple generality could be handled by *suppositio* (see e.g. Ashworth 1978). Nor do we deny that Parsons' *Linguish*, like Stoic quantification as we interpret it, relies on anaphora and variable-free quantifying. However, to us, the pertinent question seems to be how the ancient and medieval philosophers *themselves* theorized about multiple generality, and here the differences appear considerable. The Stoics' notion of a function-like predicate and their regimented representation of universals as indefinite conditionals sets them on a path of likely success, as does their concept of a regimented language itself, and their theory seems astoundingly modern and uncomplicated. By contrast, the advanced mental yoga required to express *in terms of suppositio* the reconstruction Parsons offers requires a linguistic suppleness that only a few can master, and with markedly twisted results. Moreover, the positive results appear restricted to combinations with a genitive phrase, as e.g. with 'some person's donkey', 'every person's donkey'. That said, we do not deny that the medieval and early Renaissance logicians may have had a better theoretical grasp on the problems of multiple generality than the Stoics or that there is more evidence that they discussed the question of the validity of inferences with multiply general premises. Still, a main reason why they encounter these problems in the first place is their Aristotelian term-logical conceptualization of the structure of sentences, something the Stoics did not have to face.

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